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FACTORS AFFECTING THE UNIT COST OF WEAPON SYSTEMS

THESIS

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Abstract

This research identifies variables and specifies equations that can be used to estimate the unit production cost of a weapon system. It is concerned with both explanation and prediction. Three major variables identified are cumulative quantity, production rate, and change in regime. Cumulative quantity is used in learning curve theory. Production rate is found in the U-shaped short- and long-run cost curves of economic theory. This study uses the term regime to refer to any major change in the production environment of a weapon system. This research attempts to integrate the use of these three variables.

A change in regime may be due to a change in acquisition strategy, configuration, or manufacturing method. It is recommended that a categorical variable be used to capture the effect of a change in regime. Several specific equations are proposed and discussed. In general, they entail a shift, shift and rotation, or shift and two rotations of the cost-quantity-rate surface due to a change in regime. Many accepted methods of integrating learning and rate do not produce U-shaped rate curves; this study suggests one that does. Principles and equations discussed are applied in modeling the cost history of three missile systems.

FACTORS AFFECTING THE UNIT COST OF WEAPON SYSTEMS

I. Background and Statement of the Problem

Introduction

The purpose of this study is to help identify variables and specify equations that can be used to estimate unit production cost. This thesis is concerned with both explanation and prediction. Factors that affect production cost and equations used to estimate production cost will be discussed. The statistical technique of regression analysis will be used and a basic knowledge of regression is assumed

Two major traditions shape our thinking on these topics. One is the learning curve concept. This started in the aircraft industry prior to World War II. Knowledge and familiarity with the learning curve continues to spread from the Department of Defense to academia and society at large. It can be represented through the general equation Cost = f(cumulative quantity), (Wright, 1936).

The other major tradition comes from microeconomic theory and includes the production function and cost curves for the firm. The theory of production and cost has proven useful in explaining economic behavior in many industries and settings around the world and through history. Although commonplace in Economics departments, adoption of this approach has been

sporadic in the Department of Defense. The general equation is Cost = f(production rate), (Ferguson and Gould, 1975, Mansfield, 1970).

There have been attempts to integrate these two fundamental approaches both conceptually and in terms of estimation. The general equation is Y = f(cumulative quantity, production rate), (Bemis, 1981).

In addition to these two traditions, estimators must often consider a change to the production environment of a weapon system. This thesis refers to this as a change in regime. This could be the use of multiyear procurement after several years of single year procurement (Domin, 1984). Another example is the introduction of a second producer (Cox and Gansler, 1981). Further examples include greater automation in manufacturing, the incorporation of new technology in a weapon system, or the production of a new model of a weapon system. The general model here is either Y = f(cumulative quantity, regime), or Y = f(cumulative quantity, rate, regime).

A change in regime can be captured in regression analysis by using an indicator, or categorical, variable. The use of indicator variables is a standard technique in regression analysis (Neter, 1990:455-496). However little attention has been devoted to using indicator variables when fitting learning curves, or rate augmented learning curves. This thesis will investigate the addition of indicator variables to standard models.

The following general cost models will be examined in this thesis:

(1) Y = f(cumulative quantity)
(2) Y = f(production rate)
(3) Y = f(cumulative quantity, production rate)
(4) Y = f(cumulative quantity, regime)
(5) Y = f(cumulative quantity, production rate, regime)

Two other possible models, Y = f(production rate, regime) and Y = f(regime), are remote from both economic theory and DoD cost estimating practice, and will not be discussed.

In addition to identifying and discussing variables, this study will specify and discuss equations that can be estimated with historical data. Estimation is the major focus of this thesis. Since the emphasis is on using regression analysis to analyze data, the study is largely empirical in nature. However, in order to make sense and be persuasive, the objective is to use equations that have a good conceptual basis and are consistent with prior theory and experience.

Production Cost

The Department of Defense must estimate the cost of weapon systems in order to plan, manage and budget its operations. In addition, economy of force is one of the traditional principles of war, and in the broad sense, estimation of future costs is essential to the achievement of this principle (Ely, 1997). The total cost of a weapon system is its life cycle cost. This includes all costs incurred from the inception of the weapon system to its ultimate retirement

(SCEA Glossary of Terms, 1994:140). It is often referred to as the cost from the cradle to the grave, or from the womb to the tomb.

The Army cost breakdown for life cycle cost is shown in Appendix 1, Army Cost Element Structure. It has six major categories: 1) Research, Development, Test, and Evaluation, 2) Production, 3) Military Construction, 4) Military Personnel, 5) Operations and Maintenance, and 6) Defense Business and Operations Fund. Each category is further subdivided to obtain a detailed breakdown of costs. Life cycle cost elements are mutually exclusive and collectively exhaustive (Cost Analysis Manual, 1997:148-163).

Life cycle cost is an important term and concept. However attention is often directed to one or another subset of the total cost of a weapon system. The subset chosen depends upon the objective. Costs may be estimated for near term annual budgets, for a particular contractor, or for a particular contract. Future costs, such as production costs or operating costs, may be estimated for planning purposes.

This study pertains to Recurring Production, 2.02, in the Army cost element structure. 2.01, Nonrecurring Production, is excluded from this study. Definitions for these cost elements are contained in Appendix 2, Cost Definitions. Also excluded from this study are research and development costs. References to production or manufacturing cost in this study should be taken as referring to Recurring Production.

In practice, analysts may find that costs identified in contractor or government records as recurring production may contain some non-recurring costs. However, major separately identifiable non-recurring investment will be excluded from recurring production.

To examine this issue, consider a production program that includes major expenditures for initial production facilities. Initial production facilities (IPF) is defined in appendix 2 and is classified as non-recurring production. If separate IPF contracts are let for each year's expenditure, these non-recurring expenditures will be mapped to 2.01. However, some non-recurring costs may still remain in 2.02. The recurring production contracts will contain overhead costs. At least some of these overhead costs will be fixed or semi-fixed and will not vary directly with production. Conceptually these are non-recurring costs (Nelson and Balut, 1996).

Even direct manufacturing labor costs may have some components that are non-recurring. The time to set-up for production depends on the number of set-ups independently of the number of items produced following a set-up. This has been recognized for some time (Wright, 1936:124, Asher, 1956:87). More recently, the author negotiated direct manufacturing manhours for the Army's 1984 Apache helicopter contract. The contractor estimated set-up hours apart from the recurring manufacturing hours that were estimated using learning curve theory. This procedure had the effect of introducing a non-recurring, or semi-fixed, aspect to direct manufacturing manhours.

introducing a non-recurring, or semi-fixed, aspect to direct manufacturing manhours.

Cost estimates for production are often expressed in terms of unit costs. Unit costs are important for planning. A high unit cost may cause a program to be canceled, or may lead to the selection of a competing program. Estimated high unit costs may lead to a change in program quantities or production rate. In order to reduce unit cost, alternative weapon technology, manufacturing technology, or the use of alternative materials may be considered. Different acquisition strategies may be considered to reduce cost. Acquisition strategies that can be considered include multi-year production, the introduction of another production source, or component breakout, that is, buying components directly from subcontractors. High production cost estimates may also lead to cost sharing through Joint Service or international production (L'Heureux and Grant, 1995).

In addition to planning, it is desirable to estimate costs for budgeting and monitoring program progress. For these reasons, we need to estimate costs, and unit costs, by year. Estimates for annual costs should include the impact of inflation and may require consideration of changes in relative prices (Gill, 1994).

Estimating Level

This thesis discusses many models. It also discusses many estimating levels, for example labor hours and total recurring cost, or, average fixed cost and average total cost. Many models are robust and are applicable at several estimating levels. Nevertheless questions arise as to the applicability of a particular model for estimating at a particular level of detail. Hence discussions of estimating level will reoccur throughout this thesis. Highly detailed cost estimates are described as bottoms-up, grass-roots, or engineering estimates. A bottoms-up estimate may include estimates for several categories of labor, major components, purchased parts, and raw material. To these costs are added applicable overhead rates and profit. Developing such detailed estimates may require a significant amount of data and time.

Other estimates are based on highly aggregated data, for example, contract cost and quantity. These top-down estimates may be made because we lack the resources necessary to develop a bottom-up estimate, or because an independent "sanity-check" of a detailed estimate is required. Yet another reason for developing top-down estimates is to determine major factors that affect cost. This is an effort to see the forest rather than the trees.

Cost estimating relationships, CERs, are equations that include performance or physical variables, such as speed, weight, or type of material used, to predict cost (SCEA Glossary of Terms, 1994:55). CERs are often based upon data from many weapon systems and are generally considered a top down

technique. Each equation estimated in this thesis will be based upon data from a particular weapon system.

Both types of estimating, bottoms-up and top-down, are valid. This study pertains to both types of estimates but the major emphasis is on top-down estimates. In particular, the emphasis is on estimating recurring production cost for a single weapon system by using a single equation.

Research Questions

The general research question addressed by this thesis is: What is the proper identification of variables and specification of equations to estimate unit production cost? Specific research questions to be addressed are:

Question 1: Is it valid to fit a linear composite learning curve given that component learning curves are linear and of varying slopes?

Question 2: Can the addition of a production rate variable improve the learning curve equation?

Question 3: Can the addition of a categorical variable improve learning or, learning and rate, equations?

Question 4: Can a categorical variable provide useful diagnostic information?

Question 5: Should new equations of learning and rate be explored?

II. Literature Review and Discussion

Introduction

The literature review and discussion will be in four parts. The first deals with the learning curve. The learning curve estimates cost as a function of cumulative quantity. The second part discusses the economic theory of production and cost. Economic theory estimates cost as a function of production rate. The third part discusses efforts to integrate the learning curve with economic theory. The final part discusses additional specific factors that can affect unit cost, and the use of indicator variables in either a learning, or a learning and rate, context.

Learning Curve Theory

The Learning Curve. T. P. Wright is credited with identifying and recommending the use of the cost-quantity relationship known as the learning curve. His 1936 article, "Factors Affecting the Cost of Airplanes," published in the Journal of Aeronautical Sciences, remains remarkably clear, complete and insightful today, some six decades later.

In the intervening time many articles have been written about the learning curve. Yelle (1979:324-328) lists 93 articles in his bibliography. A query of the Defense Technical Information Center data base resulted in the identification of 44 articles since 1979, the year of the Yelle article.

Use of the learning curve has become an essential technique for analysts working for prime contractors, subcontractors, support contractors, and for the Department of Defense. This thesis will not discuss the entire body of learning curve theory and practice that has developed over time. However some important aspects of the learning curve will be discussed. The learning curve equation will be used to help organize the material.

The general learning curve equation is:

$$(6) \qquad Y = A\,X^{b}$$

Y is cost. X is cumulative quantity. The exponent, b, is a small negative number and determines the rate of cost reduction, or the learning rate. The coefficient A is the theoretical cost of the first item produced.

The learning curve equation started primarily as an empirical generalization. It was observed that cost falls with an increase in quantity. The learning curve equation produces a curved relationship. Cost decreases at a decreasing rate[1]. Costs are relatively high initially, fall quickly at first, and then fall at an ever slower rate. As X approaches infinity, Y approaches zero. As a practical matter, cost appears to level off during the production of a weapon system. The learning curve is non-linear when plotted in arithmetic space. When plotted in log-log space it is a straight line. A learning curve (A=100, b=-.152) is shown in Figures 1 and 2.

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[1] The first derivative, is negative: $f' = b\,A\,X^{(b-1)} < 0$. The second derivative is positive: $f'' = (b-1)\,b\,X^{(b-2)} > 0$.

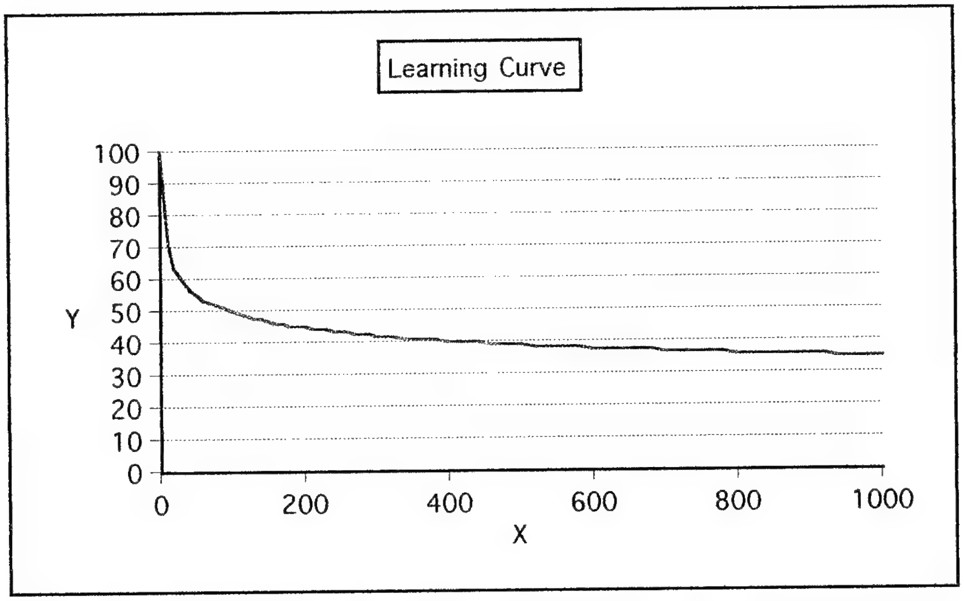

Figure 1. Learning Curve Plotted in Arithmetic Space

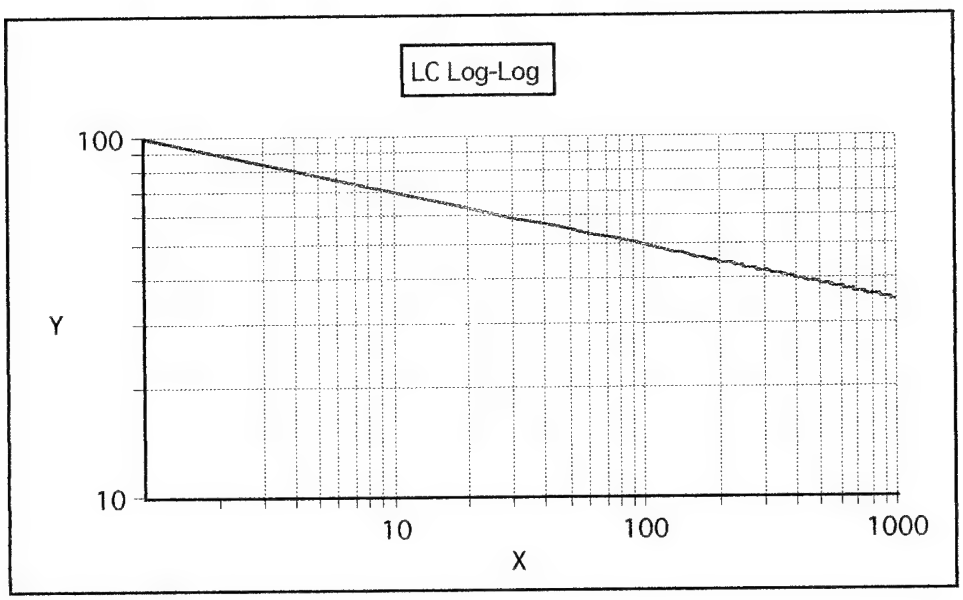

Figure 2. Learning Curve Plotted in log-log Space

X is cumulative quantity; often called the unit number. Although sometimes people omit the word cumulative, it is an essential aspect of the learning curve. The learning curve equation applies to cumulative quantity since program inception. The clock does not reset with a new year or a new contract.

The learning curve relationship is determined by raising X to the b power. The exponent b determines the rate of cost reduction associated with increasing quantity. The equation for b is:

(7) $b = \ln(\text{learning curve slope})/\ln(2)$, where the learning slope is expressed in decimal form.

The nature of the learning curve is that cost falls at some constant percentage in response to a constant percentage increase in cumulative quantity. This is normally presented in terms of a doubling, that is a 100% increase, of quantity. If cost falls ten percent with a doubling of quantity, we speak of a 10% learning rate, or a 90% learning slope (Kankey, 1983:17). Using the above equation: $b= \ln(.9)/\ln(2) =.-152$. As mentioned earlier, b is a small negative number. A range of learning curve slopes from 70% to 100%, corresponds to a range of b from -.51 to zero.

Considering the cost-quantity phenomenon chronologically, the cost of the first unit is A. "A" is also essential to the mathematical expression of the learning curve. If A were zero, cost would be zero. As we increase or decrease values of A, we raise or lower the entire learning curve.

Unit and Cumulative Average Formulations. Wright's original formulation of the learning curve is known today as the cumulative average formulation. In the cumulative average formulation, Y represents the average cost of the first X units. Using a 90% slope and a first unit cost of 100, the average cost of the first two units is 90. The average cost of the first four units is 81. The average cost of the first eight units is 72, and so on.

The first major alternative to the cumulative average formulation came from a reappraisal of Y, cost. This led to the unit formulation of the learning curve, which is sometimes called the Boeing or the Crawford curve. Using the unit theory formulation, Y is the cost of a particular unit. Once again assume a 90% slope and a first unit cost of 100. The cost of the second unit is 90. The cost of the fourth unit is 81, and the cost of the eighth unit is 72.

Given either formulation, one can calculate unit, average, and total cost. However the two formulations are distinct. One should no more mix the unit and cumulative average formulations than compute change using yen and dollars.

The cost analyst often starts with data and fits a learning curve. The data is in terms of ordered pairs: X and Y, cumulative quantity and cost. In the case of the unit curve, each data point is distinct and stands on its own. However in the cumulative average formulation, a data point includes information from prior data points. This can be demonstrated through an example. Suppose the average cost of the first four units is 81, and the average

cost of the first eight units is 72. These two bits of information are not independent. We know that if the cost of the first four units were to increase, the average cost of the first eight units would likewise increase.

The cumulative average formulation tends to smooth the data through the computation of averages. Smoothing is often used in forecasting. However the process of fitting a single learning curve to many data points also represents a way of smoothing out random variation in the data while capturing the essential connection between cost and cumulative quantity. The primary method of fitting learning curves is to use regression analysis. The cumulative average data points are correlated with one another. This suggests the possibility of serially correlated error terms in the fitted regression which is considered a problem in regression analysis (Neter et al, 1990:497-500). Statisticians generally take data points to be independent of one another and not functionally related. Therefore, even if we had a method of fitting the data other than regression analysis, there might be a statistical problem associated with the cumulative average formulation.

Ilderton (1970:13) discusses a 1969 DCAA report presented to Congress on DCAA and contractor use of learning curves. The report indicated that 93% of the curves examined were based on the unit curve formulation. Although both formulations are generally considered acceptable, this thesis will use unit theory when fitting learning curves to actual data.

Calculation of Midpoints. The revised definition of Y that came with the introduction of the unit formulation necessitates a redefinition of X. Using the cumulative average formulation, X equals 100 when the estimator has the average cost for the first 100 units. However using the unit formulation, X equals the midpoint of the lot that starts with unit 1 and ends with unit 100. The midpoint, M, is the unit whose predicted cost equals the average cost of the lot. The necessity of setting X equal to the midpoint is a bad news - good news situation. The bad news is that coming up with the lot midpoint is not so simple. The good news is that we have powerful, inexpensive computers, and techniques have been worked out. Three equations are provided below:

Exact solution:

(8) M = (the sum from X=F to X=L of X^b/(L-F-1))^(1/b)

Approximation 1:

(9) M = (((L+.5)^(b+1)-(F-.5)^(b+1))/((b+1)*LotSize)) ^(1/b)

Approximation 2:

(10) M = (F+L+2*$(F*L)^{.5}$) /4

In each case, M = lot Midpoint, F= First unit of lot, L = Last unit of lot. These equations apply to any lot with any first or last unit. The first two methods recognize that M depends on the value of b and, implicitly, the learning curve slope. The derivation of equations 8 and 9 are contained in Appendix 3, Midpoint Equations. An Excel macro function for calculating the exact solution, equation 8, is provided in Appendix 4, Exact Midpoint

Calculation. The third method calculates M independently of b. It uses the average of the arithmetic and geometric means (Nussbaum, 1994:3). The two approximation equations can be incorporated easily in an electronic spreadsheet or other personal computer software.

Results using the three approaches for calculating lot midpoints are shown in tables 1 and 2 below. The last two columns show the absolute value of the error expressed as a percentage.

Table 1. Lot Midpoints Using a 90% Learning Curve Slope

First	Last	M exact	M Apx 1	M Apx 2	Abs % err 1	Abs % err 2
1	10	4.36	4.32	4.33	0.94%	0.67%
11	20	15.19	15.18	15.17	0.02%	0.13%
21	30	25.31	25.31	25.30	0.01%	0.05%
31	40	35.37	35.36	35.36	0.00%	0.02%
41	50	45.40	45.39	45.39	0.00%	0.02%
				Avg error=	0.19%	0.18%

Table 2. Midpoints Using a 70% Learning Curve Slope

First	Last	M exact	M Apx 1	M Apx 2	Abs % err 1	Abs % err 2
1	10	3.95	3.88	4.33	1.83%	9.65%
11	20	15.09	15.08	15.17	0.03%	0.53%
21	30	25.25	25.25	25.30	0.01%	0.19%
31	40	35.32	35.32	35.36	0.01%	0.10%
41	50	45.36	45.36	45.39	0.00%	0.06%
				Avg error=	0.38%	2.10%

Differences between methods are greatest when we have a steep learning curve, 70%, and when we are in the early portion of the curve, lot 1.

Both approximation methods work well for the second and subsequent lots. Deviation from the exact solution is greatest for the first lot for both approximation methods. This is due to the rapid changes that occur early in the learning curve.

The first approximation method generally comes closer to the exact solution than does the second method. This is especially evident when we exclude the first lot. The first approximation method is also considerably more accurate when the true learning curve slope is 70%. The is due to the fact that the first approximation method takes the learning curve slope into account and generates revised lot midpoints. This thesis will use the first approximation method, equation 9, for calculating lot midpoints.

Fitting the Learning Curve. Raw data does not provide learning curve equations. It is seldom the case that one has the cost of the first unit, and one must always estimate or assume a value for "b".

Contract information is available within a project office that allows the analyst to treat each production contract as a production lot. One needs only the cost, quantity and sequence of each contract. Sometimes information on production lots within a contract is available. One uses lot information to estimate, or fit, a learning curve.

Using production lot data, one computes the first and last item, and average cost for each lot. Average cost should be expressed in terms of either constant dollars or manhours depending on the level of detail at which one is

estimating. Using the first and last values, one computes the midpoint for each lot by using either equation 8, 9, or 10. At this point, ordered (X,Y), (quantity, cost) pairs exist that are suitable for use in estimating the unit learning curve equation.

A common method of fitting the learning curve is to transform the learning curve equation, $Y = A X^b$, by taking the logarithm of both sides. This yields:

$$(11) \quad \ln(Y) = \ln(A) + b\ln(X)$$

Reexpressing ln(X) as x, and ln(Y) as y, regression analysis can be used to find the linear relationship between x and y. This results in the estimated equation: $y = a + bx$. The estimated coefficient "a" corresponds to our original Ln(A). Therefore $e^a = A$. The estimated coefficient b corresponds to b without transformation. Equations (9) and (10) use an assumed value for b. At this point in the procedure, an estimated value for b exists that can be used to recompute lot midpoints. This results in new X, and x, values that can be used in a new regression. One can iterate the process until the estimate for b equals the value of b used in midpoint calculations. This results in an internally consistent learning curve equation $Y = A X^b$. The fitted equation can be used for several purposes. For example, it can be used, along with planned future quantities, to estimate the cost of future contracts.

It should be noted that estimating the cost of the first unit requires extrapolation. The reason is that Y=A when X=1, but all lot midpoints are greater than 1. Regression equations go through the mean of x and the mean of y. The first unit will be some distance from the mean of x. Any misestimation of the slope, b, will have a large impact on the estimated value of "a". The effect on "A" will be even greater since $A = e^{a}$. Although A comes first chronologically, it is derived late in the estimation process. Additional data, production lots, or revisions to the data may have a large impact on A. First unit cost is often referred to as the theoretical first unit cost, or T1. The cost of the first unit is usually a mathematical abstraction rather than a measured quantity.

A Note on Logarithms. The unit formulation is a straight line in logarithmic space only. Freehand, straight line representations must be interpreted with care. For example a sketched linear learning curve that goes below the x axis usually implies an unrealistically high quantity, X. To some this may imply a negative value of y. In reality, this corresponds to a cost, Y, that is an order of magnitude less than the original scale. Y can never assume a negative value as a result of applying learning curve theory.

The Learning Curve as an Explanatory Model. The learning curve has gained increasing acceptance as an explanatory model. People learn through practice. It takes progressively fewer hours to perform a task with repetition. Individuals can reflect on their own experience to see the

merit of this argument. It takes less time to change a flat tire, or install personal computer software the second time than the first time. The greater the experience, the faster and better the task can be performed. Similarly it is reasonable to expect, and it has been observed, that manufacturing labor hours decline as workers gain experience in performing repetitive operations (Wright, 1936, Asher, 1956, and others).

Sources of Learning. Reductions in labor hours are not due exclusively to worker learning. Organizational learning is also important. Hirsch (1952:143-155) attributed 87% of labor hour savings to changes in technical knowledge which Yelle (1979:309) describes as a type of organizational learning. In the previously mentioned Apache helicopter contract negotiation, contractor personnel emphasized the contribution of industrial engineers in reducing manufacturing labor hours.

The curriculum at AFIT has emphasized numerous quantitative techniques that can reduce the inputs required to obtain a specified level of output. These techniques include line-balancing, statistical process control, total quality management, linear programming, and make-buy decisions. These techniques and others contribute to organizational learning and can lead to cost savings in labor, subcontract items, and raw material. The difference between individual and organization learning has been examined in the literature (Hirsch, 1952:143-155, Andress, 1954:89).

Asher (1956:3) identifies the following factors as leading to a decline of unit airframe cost as cumulative output increases:

1. Job familiarization by workmen, which results from the repetition of manufacturing operations.
2. General improvement in tool coordination, shop organization, and engineering liaison.
3. Development of more efficiently produced subassemblies.
4. Development of more efficient parts-supply systems.
5. Development of more efficient tools.

Many Learning Curves. The learning curve has been recognized from the start (Wright, 1936) as useful in estimating several categories of cost in addition to labor. Raw material cost can decline due to reduced scrap and rework. Purchased items include someone else's labor which declines with experience. Wright mentioned the following learning curve slopes: 80% for labor, 88% for purchased parts, 95% for raw material, and 70% for overhead (Wright, 1936:124-125). He also presents a learning curve for the complete airplane. This is a composite curve because it is based upon several separate curves. Wright's composite curve had variable slopes. It started as an 83% curve, changed slope to an 85% curve, then an 87% curve, and finally to a 90% curve (Wright, 1936:125-126).

The practice of fitting distinct learning curves to labor, subcontract items, purchased parts, and a composite curve to the entire system remains popular today. Only the practice of fitting a learning curve to overhead appears to have disappeared. Because the learning curve can be applied to cost elements other than labor, more general terms have been introduced:

progress curve, cost improvement curve, and experience curve (Badiru, 1992:176). However, this thesis uses the familiar term learning curve due to its widespread use and acceptance.

Occasionally the term learning curve is used in the restricted sense of applying to labor only. In this case, Y is usually measured in hours rather than dollars. This has the advantage of using a unit of measure that has a fixed value rather than one whose value changes over time due to inflation, deflation, or industry or company specific change in wage rate.

Analysts may fit many learning curves to labor. During the 1984 Apache Helicopter contract negotiations the author examined learning curve data for two manufacturers, Hughes Helicopter and Sikorsky, manufacturers of the Apache and Black Hawk helicopters respectively. For each company, the author grouped several types of labor (eight and fourteen respectively) into four categories: sheet metal, machining, subassembly, and final assembly. Learning curves were calculated for each category and for each company. Learning curve slopes differed by category and company; however several results were apparent. Learning was greatest for final assembly. This was followed by subassembly. Learning was flattest for sheet metal, then machining. These findings were consistent with the findings of others who have found that learning is greatest for assembly and lowest for machining (Hirsch, 1952:143-155, Hirsch, 1956:136-143, Asher, 1956:92).

The previous example came from contract negotiations. Contract negotiations and source selections provide excellent opportunities for obtaining detailed cost information. During source selection on another program, the author was shown detailed manhour data to substantiate claimed learning curve slopes for labor. The contractor displayed manufacturing hours for numerous small lots within each contract. However, it is usually difficult to obtain such detailed information. Analysts may have to treat each contract as a production lot, and estimate at the level of contract cost or price (cost to the government).

The cost and complexity of weapon systems has grown over time. Today many prime contractors rely greatly on subcontractors. For example, the fuselage of the Apache helicopter is obtained from a major subcontractor. Detailed cost estimating often entails fitting a learning curve to each of the major subcontractors (Apache lot 3 and lot 4 contract negotiations, Apache Program Office Estimates).

A multitude of linear learning curves are possible. It is also possible to fit a single composite learning curve to the entire program. Should we expect the composite curve to be linear as well? Some authors (Conway and Schultz, 1959:41, Asher, 1956:70) have suggested that the answer is "No." This leads to research question 1 which is examined further in chapters 3 and 4:

> Question 1: Is it valid to fit a linear composite learning curve given that component learning curves are linear and of varying slopes?

Other Issues Addressed by the Learning Curve. The learning curve has one independent variable, cumulative quantity. However, it has been used to address many issues, or variables, in addition to quantity. For example consideration of learning as an explanatory variable suggests that a break in production should lead to some amount of forgetting. This can be modeled by a move back up the learning curve. Sometimes effort is added or subtracted from a program due to configuration changes or revised make versus buy decisions. An addition can be modeled with an additional curve starting at unit 1. When effort is subtracted from the program, the cost of the theoretical first unit cost is adjusted downward for estimating the cost of later lots. The impact of advanced planning has also been examined in learning curve theory. Yelle (1979:314-315) indicates that the effect of pre-production planning is to reduce T1, the theoretical first unit cost, and to flatten the learning slope.

Other Formulations of the Learning Curve. Yelle (1979:302-328) and Badiru (1992:176-188) both discuss several alternative formulations of the learning curve. These include the S-curve, Stanford-B model, DeJong's model, and the Plateau model. The merit of these formulations depends partially on reason: what makes sense. However merit may be determined to an even greater extent based upon experience: what works. Yelle (1979:304) states:

> The essential point is that although the log-linear model has been, and still is, by far the most widely used model, some manufacturers have found other models that better describe their experiences.

My own experience includes working with a project office that decided to use a flatter composite learning curve for later lots. This is similar to the plateau model. The use of a composite curve of progressively declining slope was recommended as early as 1936 (Wright, 1936:125-126).

One method of examining a program to determine if the learning slope has changed is proposed in Chapter III and used in Chapter IV. The method uses an indicator variable to determine if there is a statistically significant difference in slope for different lots.

Any observed change in slope could be due to the effect of an omitted variable. One variable which comes to mind is production rate. The combined learning and rate models of several authors (Balut, 1981, Nelson and Balut, 1996, Bemis, 1981) suggest that unit cost falls with increasing production rate. Cox and Gansler (1981) emphasize U-shaped rate curves. In either case, cost falls in early production lots, because of both learning and rate effects. In later years, when rate is relatively fixed, cost declines solely due to learning. The analyst who considers learning only would see a flattening of the learning curve (Cox and Gansler, 1981:36-38).

Another reason offered to explain possible flattening of the learning curve is that there could be little pressure to learn and reduce cost in later lots (Kankey, 1983:19). This would appear to have relevance to another variable, competition. Competitive pressures may encourage more cost reducing effort and steeper learning curves (Cox and Gansler, 1981). Competition is presented

as a change in regime later in this Chapter.

Because the shape of the learning curve may be affected by omitted variables, many recent studies have used multiple regression. The potential of multivariate analysis is explained by Badiru (1992:186):

> This paper has presented a comprehensive survey of univariate and multivariate learning curves. Multivariate models are useful for detailed cost and productivity analysis in many economic and production processes. With a simple bivariate mode, it may be impossible to obtain accurate estimates for the effects of the two variables involved. Consequently, it is often necessary to consider more complicated models.

Economic Theory.

Production and Cost. Subsequent material is based upon standard economic theory as presented in Microeconomic Theory by Ferguson and Gould, Microeconomics, Theory and Applications by Mansfield, and the text to AMGT 559, Life Cycle Cost and Reliability, by Gill, specifically chapter 2, The Economics of Cost Analysis. Microeconomics includes the theory of production and cost. The starting point is the theory of production. Cost curves are derived from the theory of production.

Production theory deals with the relationship between inputs and outputs. Many different kinds of inputs may be required to produce a good or service. For example, labor, raw material, capital equipment, and land may be required. Inputs may be combined in various ways. To increase production, a

firm could hire more workers and buy more raw material while keeping the amount of capital equipment and land the same. A different approach to increasing production is to invest in capital equipment and buy more raw material, while maintaining, or reducing, the size of the work force.

Isoquant Curves. Relationships between inputs and outputs are illustrated using isoquant curves. An isoquant shows the various ways inputs can be combined in order to generate a specified level of output. In principle there are as many dimensions to an isoquant as there are inputs. However since two dimensions work best on paper, two variables are usually chosen for the purpose of illustration.

A common approach is to choose one input that tends to be relatively fixed, such as land, and one that can be varied more easily, for example labor. Land, buildings, and capital equipment are inputs that tend to be fixed in the short run. In contrast, the amount of labor utilized, raw material or energy consumed is more readily changed.

The approach taken here is to label one axis as fixed and the other as variable. One can think of "fixed" as referring to a particular input, or to a composite of several inputs. Similarly "variable" can stand-in for any, or all, of the inputs whose use can be quickly increased or decreased. Given enough time, the long run, all inputs are variable. Four isoquants are shown in Figure 3.

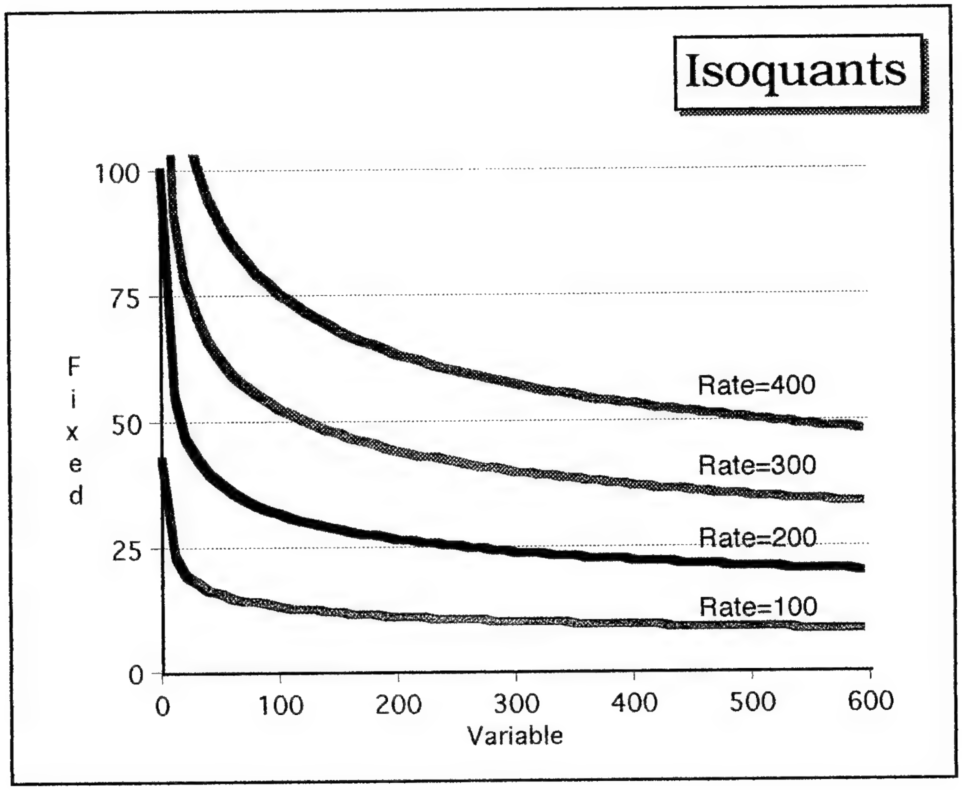

Figure 3. Isoquants

The points on an isoquant curve correspond to one level of output. Output is frequently labeled as quantity, but should really be understood as rate because production takes place in a given interval of time. The isoquants above are for the output of 100, 200, 300 and 400 units per time period.

A production function relates the use of inputs to the production of outputs. The Cobb-Douglas production function was used to produce the isoquants shown in Figure 3. Equation 12 is the equation for the Cobb-Douglas production function.

(12) $R = A\, I_1^{a1}\, I_2^{a2}$

R is rate, the level of output per period of time. I_1 and I_2 are the amount of input 1 and input 2 that are utilized. In this case, $R = 5\, F^{.2}\, V^{.8}$, with values of R set to 100, 200, 300 and 400.

Isoquants show technological possibilities. One input can be substituted for another while producing a constant level of output. Adding units of F or V, increases rate and results in a different isoquant.

If we add more of input 1 while keeping input 2 constant, rate increases. If we keep adding input 1, rate continues to increase, but does so at a decreasing rate. This is the empirically derived law of diminishing marginal returns. In the extreme, rate may decline with too much of one input. However, this would be outside of the economic region of production that we are considering.

Isocost Curves. Up to this point there has been no discussion of cost. Cost is determined by the price and quantity of the inputs used. In the two input model, the cost of production is $(I_f x P_f) + (I_v x P_v)$. A curve that has constant cost is referred to as an isocost curve. It is a straight line. In this case, it has a slope of P_v/P_f. Isocost curves that are closer to the origin have lower costs.

In order to produce a given rate at the lowest possible cost, we select the isocost line that is tangent to the appropriate isoquant curve. The least cost combinations of inputs is obtained at the point of tangency of the two curves.

Figure 4 adds isocost curves that are tangent to the isoquants previously displayed.

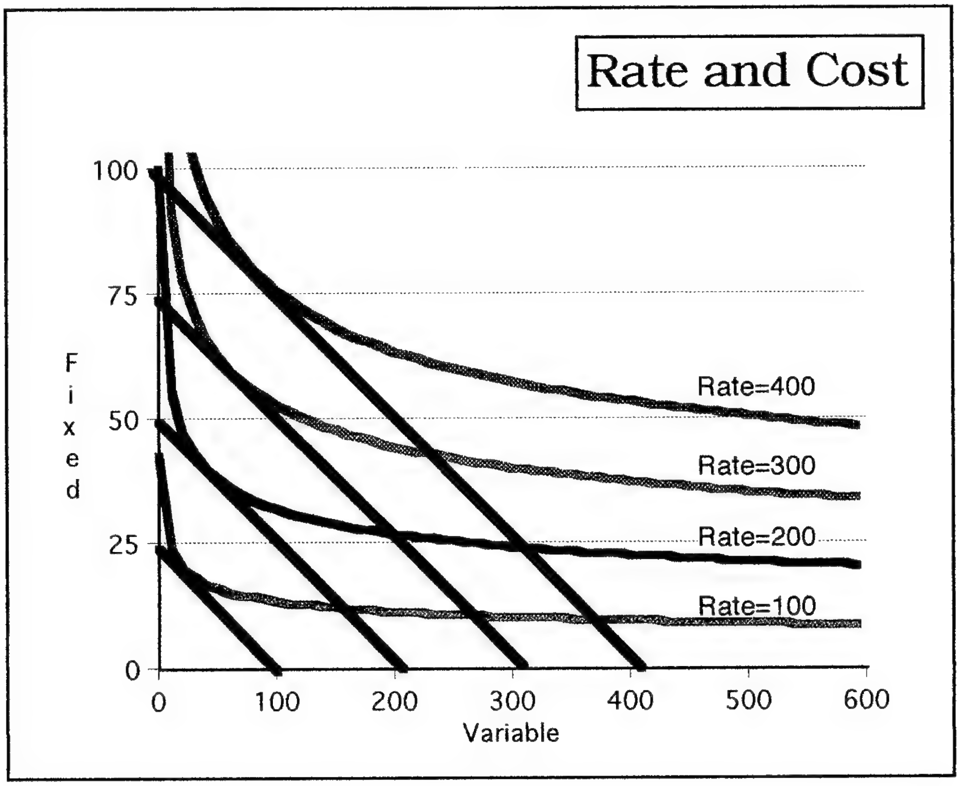

Figure 4. Least Cost Combinations of Inputs

<u>Returns to Scale; Costs in the Long Run</u>. What happens to average cost as we increase rate? Cost doubles with a doubling of inputs. If the exponents of the Cobb-Douglas production function sum to 1, as here, a doubling of inputs leads to a doubling of rate. Since cost and rate both double, average cost remains the same. This is referred to as constant returns to scale.

If the sum of the exponents is larger than one, a doubling of inputs more than doubles rate. Average cost goes down. This is referred to as economies of scale. Technological factors are one major source of economies of scale. Producing at a higher rate may enable better plant layout. Personnel and machines may mesh better at higher production rates (Ferguson, 1975:208). Certain geometrical relationships such as the relationship between area and volume may lead to more efficient production (Mansfield, 1970:138). Capital equipment may be indivisible. For example, one cannot have half of an open hearth furnace (Mansfield, 1970:138). Demand for products may become more stable at higher production rates (Mansfield, 1970:138). A second major source of economies of scale is the specialization of labor (Ferguson, 1975:208) that is possible at higher production rates. The specialization of labor due to increasing rate was first recognized by Adam Smith in The Wealth of Nations published in 1776.

If the sum of the exponents is less than one, we have diseconomies of scale. A doubling of inputs, and cost, will lead to less than a doubling of rate. Average cost increases. Diseconomies of scale have been attributed to the difficulty of coordinating large scale enterprises (Mansfield, 1970:138), or limitations in efficient management (Ferguson, 1975:208). If communication and decision making costs grow exponentially with personnel, diseconomies of scale could result. The coordination of competing product lines and possible cannibalization of sales is an issue faced by some large companies that is not

an issue for small companies. For example, International Business Machines has lost mainframe sales due to the sales of its own personal computers.

In the long run costs initially decline due to economies of scale. This is followed by a region of constant returns to scale. As a firm becomes larger and produces at a higher production rate, diseconomies of scale set in. This pattern produces a U-shaped long run average cost curve (Ferguson, 1975:198-202, Mansfield, 1970:172-175, Gill, 1994:26-29).

Standard texts on economics present references to U-shaped long run cost curves in chapters on the cost curve of the firm, competition, monopolistic competition, and oligopoly. The chapter on monopoly will usually mention several kinds of monopoly. One is the "natural monopoly." This occurs if the long-run cost curve either does not turn up, or does not turn up at a production rate relevant to the industry. The long run cost curve declines and then levels off in the relevant range (Ferguson, 1975:262, Mansfield, 1970:177).

Costs in the Short Run. The discussion so far assumes that all inputs can be varied. This is the case in the long run. However, in the short run some inputs are fixed. Holding these constant often denies the flexibility needed to choose the least cost combination of inputs. These relationships are shown in Figure 5.

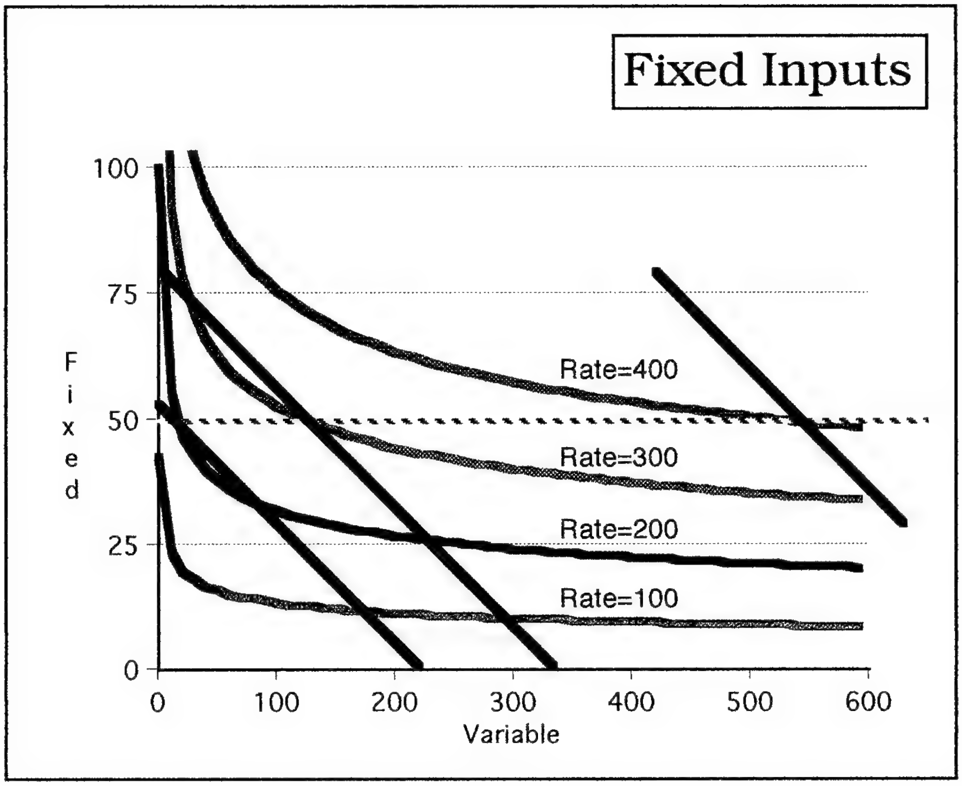

Figure 5. Production in the Short Run

Figure 5 depicts a situation where the fixed-type inputs are set at 50. Isocost lines have been added for R=200, 300 and 400 given the new constraint, F=50. At each rate, the isocost line has moved to the right compared to Figure 4, which means that cost is higher for each rate. Cost is higher for each production rate shown. 200 units of output could be produced at a lower cost, if less than 50 units of the fixed input could be utilized. If more than 50 units of the fixed input could be used, production at rates of 300 or 400 could proceed at lower cost. The greatest cost penalty occurs at production rates of 100 and 400.

Average cost depends on factor proportions. Optimal proportions are indicated by the point of tangency of isocost and isoquant curves. Given these proportions, average cost is at its minimum. Any deviation from optimal proportions, leads to higher average cost.

In the short run, there is initially too much fixed input given the low production rate. As the rate increases, average cost declines because we are making better use of our fixed input. At some point average cost achieves a minimum because the optimal mix of fixed and variable inputs is achieved. As production continues to increase, average cost increases because there is too much of the variable input and not enough of the fixed input. The short run average cost curve is U-shaped (Ferguson,1975:190-198, Mansfield, 1970:172-175, Gill, 1995:26-29).

There is a short run cost curve associated with fixed input equal to 50. If we specify a different level of fixed input, for example 75, we obtain a different short run cost curve. There are many short run cost curves but only one long run curve.

These same relationships can be developed by reference to average fixed and average variable cost curves. The average fixed cost curve declines with rate because the fixed cost is divided by an ever greater production rate. The average variable cost curve increases with rate due to the diminishing marginal return of the variable inputs. The average total cost curve is the sum of average fixed and average variable curves. Initially the reduction of

average fixed cost dominates, and the average total cost curve declines. At higher production rates, the increase in average variable cost dominates and the average total cost curve increases.

In the long run all inputs can be varied. In the short run some inputs can be varied. Because of the added flexibility, the long run cost curve is always lower than, or tangent to, any of the short run cost curves. The long run cost curve and two short run cost curves are shown below in Figure 6. The vertical axis is average cost. Sometimes it is referred to as average total cost because it equals the sum of average fixed cost and average variable cost. The horizontal axis is production rate.

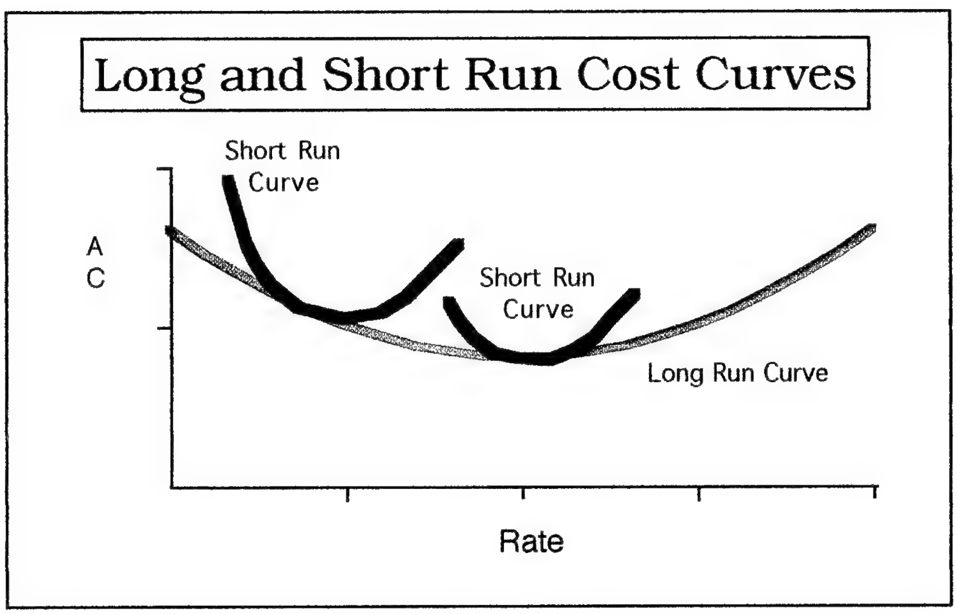

Figure 6. Long and Short Run Cost Curves

In a two input model, there are short and long run cost curves. A model that has many inputs may have medium runs, depending on the speed at which each input may be varied. However in short and medium runs, average cost curves are U-shaped. The long run cost curve is U-shaped, except in the case of a natural monopoly. In terms of estimation, one way to fit a U-shaped curve is through the use of a quadratic equation relating average cost, Y, to rate, R. This is shown below:

$$(13) \quad Y = a + b\,R + c\,R^2$$

The economic discussion to this point assumes a given set of input prices. If input prices change, the analysis must be modified somewhat. For example, if overtime pay is authorized at higher production rates, average cost will increase. The discussion to this point also assumes a given level of technology.

Technology. Mansfield (1970:10) states: "The important thing about technology is that it sets limits on the amount and types of goods that can be derived from a given amount of resources." He further states (Mansfield, 1970:118):

> For any commodity, the *production function* (italics in the original) is the relationship between the quantities of various inputs used per period of time and the maximum quantity of the commodity that can be produced per period of time. More specifically, the production function is a table, a graph, or an equation showing the maximum output rate that can be achieved from any specified set of usage rate of inputs. The production function summarizes the characteristics of existing technology at a given point in time; it shows the technological constraints that the firm must reckon with.

Ferguson (1975:124) states:

> The theory of production consists of an analysis of *how* the businessman—given the "state of the art" or technology—combines various inputs to produce a stipulated output in an economically efficient manner.

Technological advance changes isoquant curves and, by implication, cost curves. Isoquant curves change because more output can be obtained with a given amount of input. More output can be obtained at a given cost and average cost falls.

Technological change is an important concept which links standard economic theory and the learning curve phenomenon. DoD experience in the production of weapon systems indicates that cost reducing technological advance is correlated with increasing cumulative production. This is another way of stating that weapon systems undergo learning.

Efforts to Integrate Learning and Economic Theory

Introduction. Efforts to integrate learning and economic theory are more recent. Three approaches will be discussed briefly below. One distinguishes between fixed and variable costs, and estimates each separately. Another uses a production function that is modified to address learning. Yet another approach augments the learning curve equation with a rate variable.

Estimating Fixed and Variable Costs. The first approach can be found in the work of Smith (1981) and Balut (1981, Balut, Gulledge and Womer, 1989, Nelson, Balut, 1996). The driving factor in determining the production rate effect is assumed to be the reduction of average fixed cost. This is manifested in a reduction in contractor overhead rates. In Balut's approach (Balut, 1989, 1996), contractor direct costs are mapped to the economists' category of variable cost. Contractor overhead, or indirect, costs are separated and mapped to variable cost and to fixed cost. Fixed cost provides the basis for the production rate effect. As the contractor's business base grows, the overhead rate declines. Variable cost, both direct and indirect, is estimated using learning curve theory. Overhead rate factors are applied to variable cost in order to allocate fixed cost. Since this approach is based exclusively on the reduction of average fixed cost resulting from an increase in rate, it does not generate a U-shaped cost curve.

Using Production Functions. A second approach emphasizes micro-economic theory. A good discussion of major studies can be found in Giuliano (1995). Early studies include those by Alchian (1963:679-693) and Rosen (1972:366-382). More recent effort includes several contributions by Womer and Gulledge including The Economics of Made-to-Order Production (Gulledge and Womer, 1986). The studies of Womer and Gulledge, individually and in collaboration, use a production function, similar to the Cobb-Douglas function shown in Equation 12 which relates inputs to output. The production function is augmented by a learning hypothesis. Labor productivity depends upon learning which is functionally related to cumulative quantity. This is a theoretically robust approach which integrates learning curve theory with production/cost theory at a very low level. It is a general approach that does not restrict the rate effect to a negative slope. The approach has the potential of providing new insights and demonstrating new findings for both economic theory and learning curve theory.

Adding a Rate Term. Another major approach involves the direct estimation of cost using cumulative production and rate. Rate can be expressed as quantity per month, quantity per year, or as a ratio to some standard, or economic rate. Preston and Keachie (1964:100-106) estimated both labor hours and cost with a variety of estimating equations that showed cumulative production and rate to be statistically significant.

More recent practice has used a learning curve equation augmented by a rate term:

$$(14) \quad Y = A X^{b} R^{c}$$

Smith (1976) used this equation to predict labor hours. More recently, equation 14 has been applied to recurring production cost (Bemis, 1981). Bemis popularized this approach which is sometimes referred to as the Bemis model. It can also be described as a rate augmented learning curve. Equation 14 will be applied to recurring production cost in this thesis.

Taking the logarithm of both sides of equation 14 produces the following equation:

$$(15) \quad \ln Y = \ln A + b \ln X + c \ln R$$

Expressing LnY as y, LnA as a, LnX as x, and LnR as r results in an equation that can be estimated using multiple regression: $y = a + bx + cr$. As with the learning curve, iteration is required to generate an internally consistent model in which the estimated value of b equals the value of b used to generate lot midpoints.

Interpretation is similar to that of the learning curve. "A" is the theoretical cost when cumulative quantity is 1, and rate is 1. It could be referred to as T1R1, analogous to T1 in learning curve theory. As in the case of the learning curve, "A" is a theoretical value based upon extrapolating from the data set. b has the same interpretation as in learning curve theory. c has an interpretation very similar to b:

(16) $c = \ln(\text{rate slope})/\ln(2)$

Like b, the expectation is that c will be a small negative number. Regardless of sign, c cannot generate a U-shaped cost-rate curve. Equation 14 results in a three dimensional cost, cumulative quantity, rate surface. A hypothetical example with A equal to 50,000, a learning slope of 96% and a rate slope of 94% is depicted below:

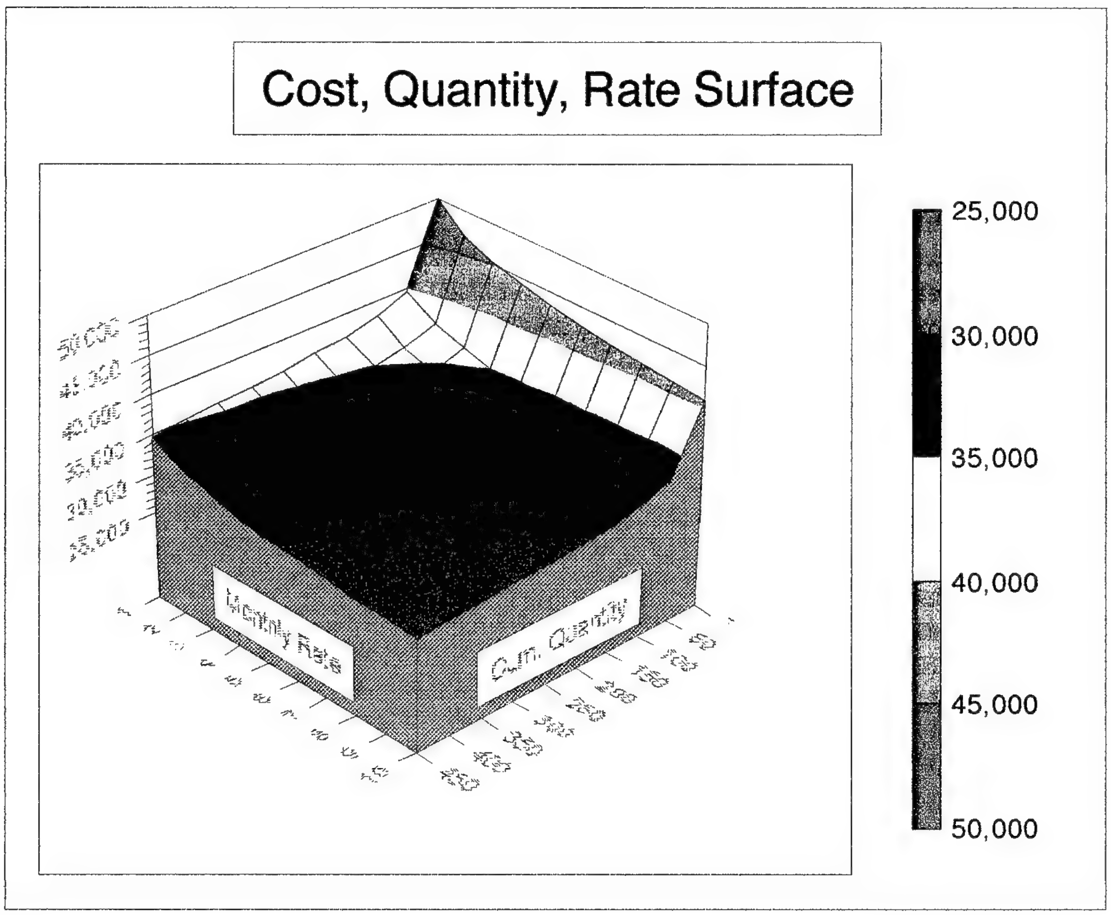

Figure 7. Cost, Quantity, Rate Surface

An advantage of this approach is that annual and cumulative quantity, and average lot cost are data that are readily available. Average lot cost and cumulative quantity are the basic inputs for deriving a learning curve. Annual quantity can be used as a proxy for rate. One can also divide annual quantity by 12 to express rate as a monthly value.

Determining Rate. Although annual quantity is often a reasonable proxy for rate, analysts may encouraged to examine monthly production and delivery schedules if that data is available. Doing so may lead to new insights and a different value for the rate variable. Annual contracts may not correspond to 12 months of effort. Early Apache helicopter contracts required about 30 months of effort. There was considerable overlap between contracts and the contractor often worked on three successive annual buys at once. There may also be breaks between contracts. At a minimum, one should not be surprised by a Holiday break at the end of the year.

However, it should be noted that overlaps or breaks between contracts are complicating factors for learning theory too. Overlaps between contracts would seem to deny latter contracts the full learning benefit of prior contracts. Breaks would suggest the possibility of lost learning, or forgetting.

Multicollinearity. Another problem stems from the fact that cumulative quantity and rate are often correlated. The author has observed that weapon system production often follows a general pattern. This same pattern was described by the instructor for the ACEIT model (Cost 674, Seminar

in Cost Analysis). The pattern is for production to have three phases. They are, in chronological order: 1) production rate increases, 2) production rate levels off, and 3) production rate declines. Production rate increases as numerous production issues are resolved. Production continues at the planned economical rate. Production rate declines and enables an orderly transition to other work. Production ramps up, levels off, and then declines. This contrasts with cumulative quantity which always increases. This is illustrated in Figure 8.

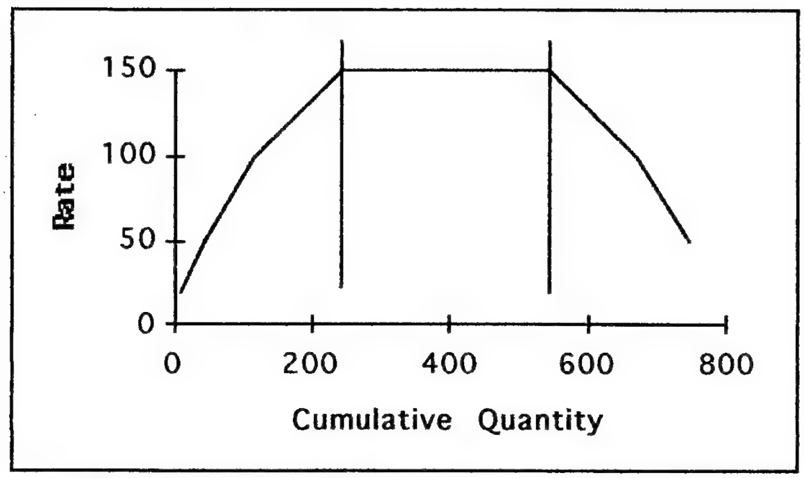

Figure 8. Rate and Cumulative Quantity

If the data is in the first phase, rate and cumulative quantity will be highly correlated. When X and R are highly correlated, the regressors x and r are highly correlated also. In regression analysis this situation is called multicollinearity.

Multicollinearity is a problem in regression analysis (Neter et al, 1990:285-295, 385-388). When independent variables move together, the multiple regression algorithm has difficulty sorting out their individual contributions. This may result in unreliable estimates for the regression coefficients.

> The fact that some or all predictor variables are correlated among themselves does not, in general, inhibit our ability to obtain a good fit nor does it tend to affect inferences about mean responses or predictions of new observations, provided these inferences are made within the region of observations. (Neter et al, 1990:289)

Another sign and consequence of multicollinearity is that adding or deleting variables changes the regression coefficients of the remaining variables (Neter, 1990:385). Estimated regression coefficients may have an algebraic sign that is opposite of that expected from theoretical considerations of prior experience (Neter, 1996:385). Multicollinearity also increases the estimated standard deviations of the regression coefficients (Neter, 1990:385). This reduces the t values of the regression coefficients. This may cause some important variables to appear statistically insignificant (Neter, 1990:385).

Extensions. The Bemis equation, $Y = A\,X^b\,R^c$, is an extension of the learning curve equation, $Y = A\,X^b$. These equations can be extended further. For example, Moses (1996), building on the work of Greer and Liao (1987), examines several equations including: $Y = A\,X^b\,R^c\,CR^d\,IR^f$. CR is company-wide activity rate and IR is industry capacity utilization rate. In Chapter III,

extensions to the unit formulation and Bemis models are proposed. These extensions use an indicator variable to capture the effect of a change in regime.

A Change in Regime.

Introduction. A weapon system may undergo a change in its production environment. In this thesis, this is referred to as a change in regime. The regime may change because of a change in acquisition strategy, manufacturing method, or a change in the configuration of the weapon system. Examples of changing acquisition strategy are the use of multiyear procurement, dual source procurement, and component breakout. Multiyear production and competition will be discussed in detail below. Increased use of automation is an example of a change in manufacturing method. There are many ways that the configuration of a weapon system may change. A series of changes is often captured with new model designators, "A" model, "B" model, etc.

In order to evaluate the impact of a change in regime, the analyst must separate the impact of learning and regime. If a learning and rate model is used, the estimator must evaluate the separate impacts of learning, production rate, and regime.

Adding Categorical Variables. A general method for estimating the impact of regime will be proposed and discussed thoroughly in Chapter III. It is based upon the use of categorical variables. Although categorical variables are a standard tool in regression analysis (Neter, 1990:455-496), the author has been unable to find any discussion in the literature that explores or emphasizes the use of categorical variables in a learning, or learning and rate, context.

An objective of this thesis is to emphasize and develop the use of categorical variables to address the impact of a change in regime in a learning, or a learning and rate, context. Specific equations for evaluating a shift, or a shift and rotation(s) of the learning curve or the cost-quantity-rate surface will be proposed in Chapter III and applied in Chapter IV.

Multiyear Procurement. Multiyear procurement, or multiyear contracting, is a method of contracting by which several annual requirements, up to 5, are obtained with a single contract. Congress funds a multiyear program on an annual basis. However, in the event of program cancellation, the contractor can recoup expenses made for future year requirements (Domin, 1984:1-1). Multiyear procurement increases program stability and extends the contractor's and subcontractors' planning horizons. Multiyear procurement allows the contractor to make commitments that might otherwise be imprudent. Cost savings from multiyear procurement have been attributed to: 1) reduced prices for parts and material, 2) avoidance of price

escalation, 3) improved efficiency by the prime contractor (Domin, 1984: ii, 2-3 to 2-4).

Cost savings from multiyear procurement have been estimated at 7.9% for the Black Hawk helicopter, and 8.9% for the F-16 compared to the use of annual contracts (Domin, 1984:2-1). GAO estimated the average savings from multiyear procurement for 12 major systems at 9.7% (GAO, 1982:10).

A typical scenario involves the introduction of a multiyear procurement after several successive single year buys. The actual multiyear cost can be compared to an estimate of what the cost would have been if single year contracts had been used instead. Single year estimates are based upon a projection of the learning curve. This is depicted in Figure 9.

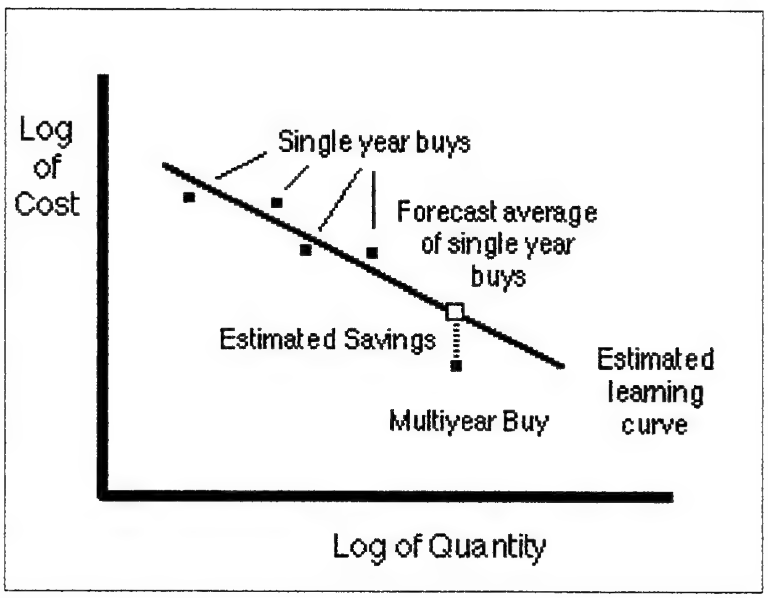

Figure 9. Single Year and Multiyear Procurement

From a learning, or learning and rate, perspective, cost savings can be described as a change in "A", the theoretical first unit cost. A method for evaluating a change in A is proposed in Chapter III.

Competition. The availability of two producers for a major weapon system, for example a helicopter or missile, or a major component, such as an engine, is usually described as competition. Dual source production contrasts with the common situation where only one producer exists. Competition may be present at the start of production, or a second source may be introduced after several sole source buys have taken place.

Cost savings attributable to competition are often considerable but have varied greatly from procurement to procurement (Washington, 1996:1-2). Flynn and Herrin (1990) examined Navy experience in the 1980's for 12 programs. Average net savings attributable to competition were 14.2% of the estimated cost of sole source production. Savings ranged from .8% to 27.9%.

Cost savings attributable to competing a weapon system must be estimated. The general approach is similar to that described for estimating multiyear procurement savings. A second source is often introduced after several sole source buys have taken place. The sole source buys are used to develop a sole source learning curve. This is used to estimate cost given continued sole source procurement. This estimate is compared to the actual cost from dual source production. After several competitive contracts have been let, learning curves can be estimated for the competitive regime. One

can then compare sole source and competitive learning curves.

Cox and Gansler (1981) explored the impact of competition adjusting for learning and production rate. They described the relation between cost and rate as U-shaped in both the short and long run (Cox and Gansler, 1981:32-35). Based on prior studies, they expected that the introduction of competition would lead to immediate cost savings, and a steeper learning curve: a shift and rotation of the learning curve. This is consistent with the view that permanent competitive pressure leads to greater learning and progressively greater percentage cost savings. A shift and rotation of the learning curve is illustrated in Figure 10.

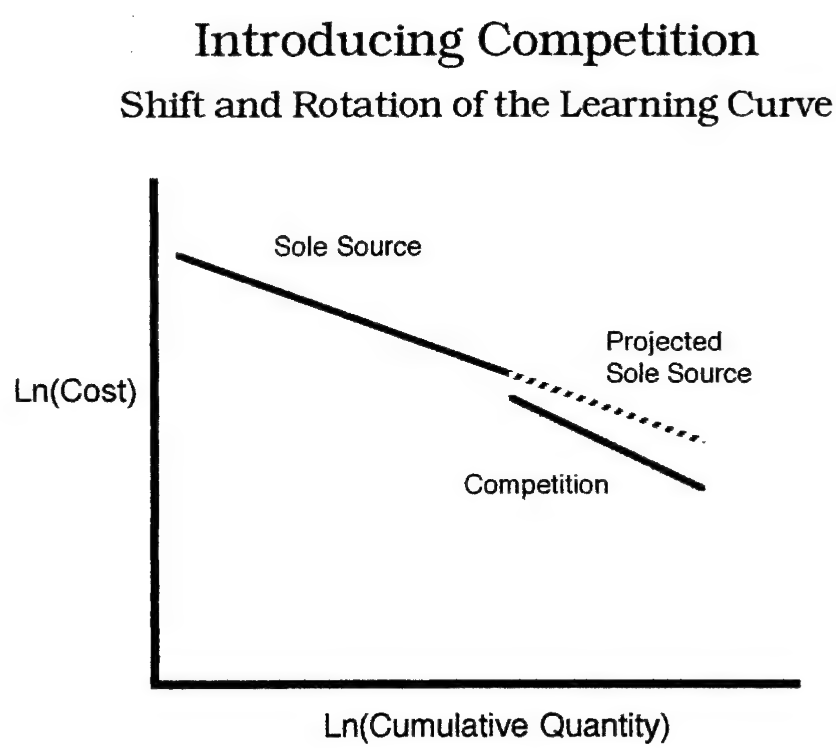

Figure 10. Sole and Dual Source Production

Cox and Gansler examined five tactical missile systems, eight military electronics programs, and one guided missile frigate. In each program, a second source was introduced after several years of sole source production. The actual dual source experience was compared to what would be expected given continuance of the sole source regime.

In the case of the frigate, the slope was 4% steeper under competition. However the first buy from the second source was more expensive than from the original source who had more experience. The learning curve of the second source was steep but involved an upward shift.

In the case of the missile systems, the learning curve slope of the second source was, on average, 5% steeper than that of the first source. The second source experienced some upward shift of the curve. Electronic systems experienced an immediate reduction in cost, a downward shift of the learning curve. There was inadequate data to estimate a dual source learning slope.

Although results varied, they were able to conclude (Cox and Gansler, 1981:42):

> For less costly and complex systems, a second source can be competitive from the outset; as cost and complexity increase, more time is required for a second source to be competitive. Furthermore, in *all cases where there was sufficient data to permit analysis, the slope of the collective cost improvement curve of the second source was steeper than that of the original producer.*

The authors examined competitive savings and the duration of the sole source regime. They observed that the later the introduction of competition, the greater the downward shift and rotation of the learning curve. They viewed this as "making up" for potential but unrealized cost savings. The authors generated an optimal learning curve based on competition starting with the first buy (Cox and Gansler, 1981:42-45). It has the original first unit cost but a steeper learning slope than experienced in sole source production.

III. Methodology

Adding Learning Curves

A prime contractor may have a separate learning curve for each labor category. A learning curve may be applied to each subcontractor. Further, learning curves, or quantity discounts, may be computed for purchased parts and raw material. To these costs, various overhead rates are applied. Given this aggregation of cost, one might question the merit of a single, simple composite weapon system learning curve.

What happens when several learning curves are added together? Does it result in a composite learning curve that could be fit directly? It has been pointed out (Conway and Schultz, 1959:41) that summing several learning curves results in a composite learning curve that is not linear in the logarithms. Asher (1956:70) states:

> If an analysis of actual data reveals that departmental progress curves do, in fact, have significantly different slopes from each other, then the unit curve (the sum of the departmental curves) cannot be linear.

This reasoning applies to an even greater degree in aggregating curves from the steepest, assembly labor, to the flattest, raw materials to come up with a recurring production cost learning curve. One might conclude that it is inappropriate to fit a composite learning curve that is linear in the logarithms. This leads to the following research question:

Question 1: Is it valid to fit a linear composite learning curve given that component learning curves are linear and of varying slopes?

A hypothetical case is examined in Chapter IV to address this important question.

Quantity and Rate

Research question 2 is reproduced below:

Question 2: Can the addition of a production rate variable improve the learning curve equation?

The theoretical basis for considering production rate was discussed in Chapter II. The effort to integrate rate effects with learning was also discussed. One approach recommended was to use equation (14) $Y= A X^b R^c$. This equation will be used further in Chapter IV to analyze the cost history of three weapon systems.

Using Categorical Variables

Introduction. The use of categorical variables is an extension of the basic learning and rate augmented learning models. Categorical variables, also known as indicator, or dummy variables, are a standard tool in multiple regression analysis (Neter, 1990:455-496). A categorical variable assumes a value of zero or one to distinguish between categories. In this thesis, categories are called regimes. The categorical variable will have a value of zero in $regime_0$, and a value of one in $regime_1$.

Adding a categorical variable for regime to the basic learning curve model leads to either a shift, or a shift and rotation, of the learning curve. Adding a categorical variable for regime to the rate augmented learning curve model leads to either a shift of the cost-quantity-rate surface, or a shift with one, or two rotations to the cost-quantity-rate surface depicted in Figure 7.

Learning and Regime. Consider a program that has been produced for many years in a sole source regime. A second source is introduced and competitive, dual source production continues for many years. There are two regimes, sole source and competitive. The categorical variable can be given a value of zero for the sole source procurements, and a value of one for the competitive procurements.

One way to specify the learning curve to account for a change in regime is proposed below:

$$(17) \quad Y = A\, e^{cZ} X^{(b+dZ)}$$

Z is the categorical variable which assumes a value of zero or one. "e" is the base of the natural logarithms. When Z has a value of zero, equation 17 reduces to the standard learning curve equation. When Z has a value of one, we obtain a revised first unit cost, Ae^{c}, and a revised learning coefficient, b+d. Taking the logarithm of both sides results in:

$$(18) \quad \ln(Y) = \ln(A) + cZ + (b+dZ)\ln(X)$$

Expressing ln(Y) as y, ln(X) as x, and ln(A) as "a" results in the following equation: $y = a + cZ + (b+dZ)x$, or , $y = a + cZ + bx + dZx$. Expressing Zx

as the variable Q, results in the equation $y = a + cZ + bx + dQ$. This can be estimated using multiple regression and results in the estimated coefficients a, b, c, and d. Raising e to the "a" power yields the theoretical first unit cost for $regime_0$: $e^a = A$. For $regime_1$, the theoretical first unit cost is equal to e^{a+c}. For $regime_0$, b is interpreted in the normal way. The corresponding value for $regime_1$ is equal to (b+d).

This procedure results in a shift and rotation of the learning curve. In other words, it results in separate learning curves for $regime_0$ and $regime_1$. The identical curves could be fit by simply breaking the data into two data sets and fitting each separately. Some change in learning curve intercepts and slopes is anticipated due to random variation in the data but these measured change may or may not be statistically significant. An advantage of the proposed method is that it provides additional diagnostic information. Tests for the c and d coefficients will indicate if the change in intercept and slope are statistically significant.

There are occasions where one expects the intercept to vary but the slope to remain the same for the two regimes, a shift of the learning curve. An example discussed in chapter 2 is the use of multiyear contracts after several years of single year contracts. Equation 19 can be used to evaluate a shift of the learning curve.

$$(19) \quad Y = A\, e^{cZ} X^b$$

When transformed, we obtain:

$$(20) \quad \ln(Y) = \ln(A) + cZ + b\ln(X)$$

In $regime_0$, single year procurement, this reduces to the standard learning curve. In $regime_1$, multiyear procurement, a revised first unit cost is calculated, Ae^{cZ}. If multiyear procurement results in savings as expected, the estimated coefficient c will be a negative number. Multiyear savings, accounting for learning and expressed as a percentage, will be:

$$(21) \quad \text{Multiyear Savings Percent} = 100*(Ae^{cZ}/A)$$

It is possible to hold the theoretical first unit cost fixed while allowing the learning slope to vary by regime:

$$(22) \quad Y = A\,X^{(b+dZ)}$$

Taking the logarithm of both sides:

$$(21) \quad \ln(Y) = \ln(A) + (b+dZ)\ln(X)$$

Although equation 21 can be readily estimated, this method is discouraged. Generally when the learning slope changes, the intercept is expected to change as well.

<u>Learning, Rate and Regime</u>. A categorical variable for regime can be added to equation 14, the standard rate augmented learning equation. If the expectation is that only the intercept will vary by regime, a shift of the cost-quantity-rate surface, the following equation can be used:

$$(22) \quad Y = A\,e^{cZ}\,X^b\,R^f$$

The transformed equation is:

$$(23) \quad \ln(Y) = \ln(A) + cZ + b\ln(X) + f\ln(R)$$

These equations might be appropriate in examining the impact of multiyear procurement in a rate augmented learning framework.

If we believe that the intercept and learning slope should both change, a shift with one rotation of the cost-quantity-rate surface, the following equations can be used:

$$(24) \quad Y = A\, e^{cZ}\, X^{(b+dZ)}\, R^{f}$$

$$(25) \quad \ln(Y) = \ln(A) + cZ + (b+dZ)\ln(X) + f\mathrm{Ln}(R)$$

This could be used to examine the impact of competition, or a change in technology.

A change in regime could affect the sensitivity of cost to rate in addition to learning. This would correspond to a shift with two rotations of the cost-quantity-rate surface. A change in regime from a labor intensive to a capital intensive operation would have several implications. Organizational learning would continue but the opportunity for labor learning would be less. Therefore one would expect the learning slope to be less steep. Increased automation may lead to an increase in fixed cost. One reason for the rate effect is the reduction of average fixed cost as rate increases. Therefore an increase in fixed cost should increase the rate effect. To summarize, one might anticipate that the effect of introducing capital intensive procedures would be to reduce the sensitivity of cost to learning, and increase the sensitivity of cost

to rate. The impact to the intercept, T1R1 is uncertain, and one might want to allow it to vary also. The relevant equations are:

$$(26) \quad Y = A\, e^{cZ} X^{(b+dZ)} R^{(f+gZ)}$$

$$(27) \quad \ln(Y) = \ln(A) + cZ + (b+dZ)\ln(X) + (f+gZ)\ln(R)$$

Other combinations are possible, for example one could allow the intercept and rate slope to vary by regime while keeping the learning slope constant. It is recommended that the analyst first consider the implications of a particular change in regime and then specify an equation, rather than to specify and fit all equations.

Research questions 3 and 4 are reproduced below:

Question 3: Can the addition of a categorical variable improve learning or, learning and rate, equations?

Question 4: Can a categorical variable provide useful diagnostic information?

The discussion to this point suggests an affirmative answer to questions 3 and 4. The cost history of three weapon systems will be analyzed in chapter 4 to further address questions 3 and 4.

Another Approach to Quantity and Rate

Research question 5 is reproduced below:

Question 5: Should new equations of learning and rate be explored?

The rate augmented learning curve equation has been specified as $Y = AX^b R^c$. This is the standard learning curve equation with an added rate term. It reflects the learning curve tradition modified by the addition of a rate variable.

What might happen if an economist, steeped in the economic tradition which emphasizes U-shaped cost curves, were to decide to modify standard economic theory to incorporate a rate term? The author suggests that the following specification might ensue:

$$(27) \quad Y = A + bR + cR^2 + d\mathrm{Ln}X$$

This is a quadratic, U-shaped, relationship between cost and rate with the addition of a learning term. Using this equation, the U-shaped curve would fall with increasing values of X, cumulative quantity. Costs would decrease at a decreasing rate with cumulative experience. An example is shown in Figure 11.

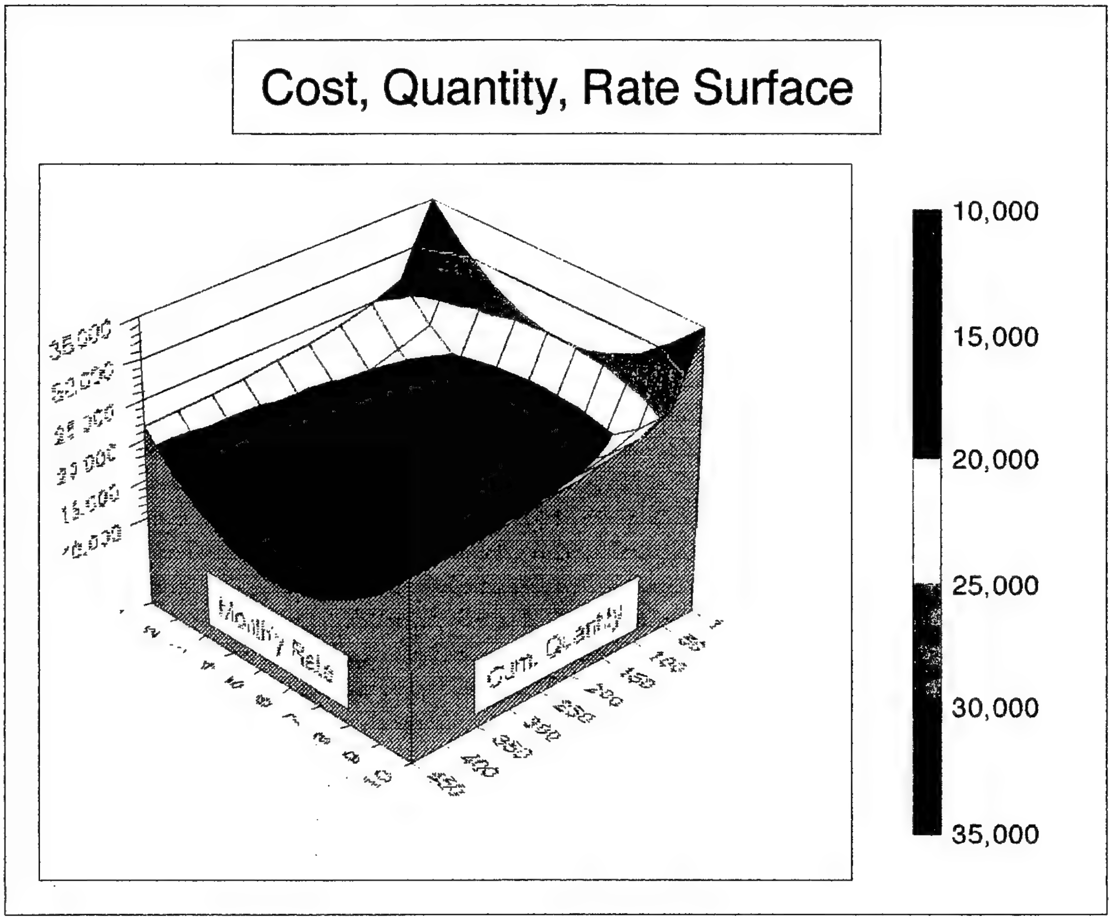

Figure 11. Alternative Cost, Quantity, Rate Surface

The advantage of this model is that the relationship between cost and rate corresponds to standard economic theory. Costs initially decline, reach a minimum, and then increase. The reasons and terminology differ based upon whether we are examining the short run, where some factors of production are fixed, or the long run, where all factors are variable. The short run discussion addresses reductions is average fixed cost, and increases in average variable cost. Short run costs pertain to whether we have too much, the right amount, or too little fixed inputs for a specified rate of production. Long run

discussions are in terms of economies of scale, constant returns to scale, and diseconomies of scale.

Cost declines over time due to cost reducing technological change, labor learning and organizational learning. This cost reduction is a function of the cumulative number of items produced.

The relationship between cost and cumulative quantity changes somewhat compared to standard learning curve theory. A doubling of quantity leads to a fixed reduction of cost, instead of the usual fixed percentage reduction of cost. This may seem strange to analysts immersed in the learning curve tradition. However the equation $Y = A + bR + cR^2 + dLnX$ should not be compared to the learning curve equation $Y = A\ X^b$, but to the rate augmented learning curve equation, $Y = A\ X^b\ R^c$. We have somewhat less experience in the simultaneous fitting of learning and rate, than in fitting a simple learning curve. The correct specification of the general equation (3) Y = f(cumulative quantity, production rate) is not certain.

The discussion in this section is highly preliminary. Equation (27) $Y = A + bR + cR^2 + dLnX$ will be applied to three weapon systems in Chapter IV. The merits of this equation is, in part, an empirical question. Given that the equation is reasonable and logical, does it work?

IV. Data Description and Analysis

Adding Learning Curves

Research question 1 is reproduced below:

Question 1: Is it valid to fit a linear composite learning curve given that component learning curves are linear and of varying slopes?

A hypothetical case will be examined to help to answer this question. Assume three cost categories, labor, subcontract items, and material. Each has its own learning curve. The learning curve for labor is based on a first unit cost of 50 and a 75% learning curve slope. The learning curve for subcontract items has a first unit cost of 35 and an 85% learning slope. The first unit cost for materials is 15 and it is on a 95% learning slope. Based upon these three inputs, costs were computed for unit 1, unit 10, and every tenth unit thereafter until unit 1,000. These were then summed to obtain the total cost for every tenth item. Four cost curves, labor, subcontract items, raw materials, and a total, or composite, cost curve are shown in Figure 12.

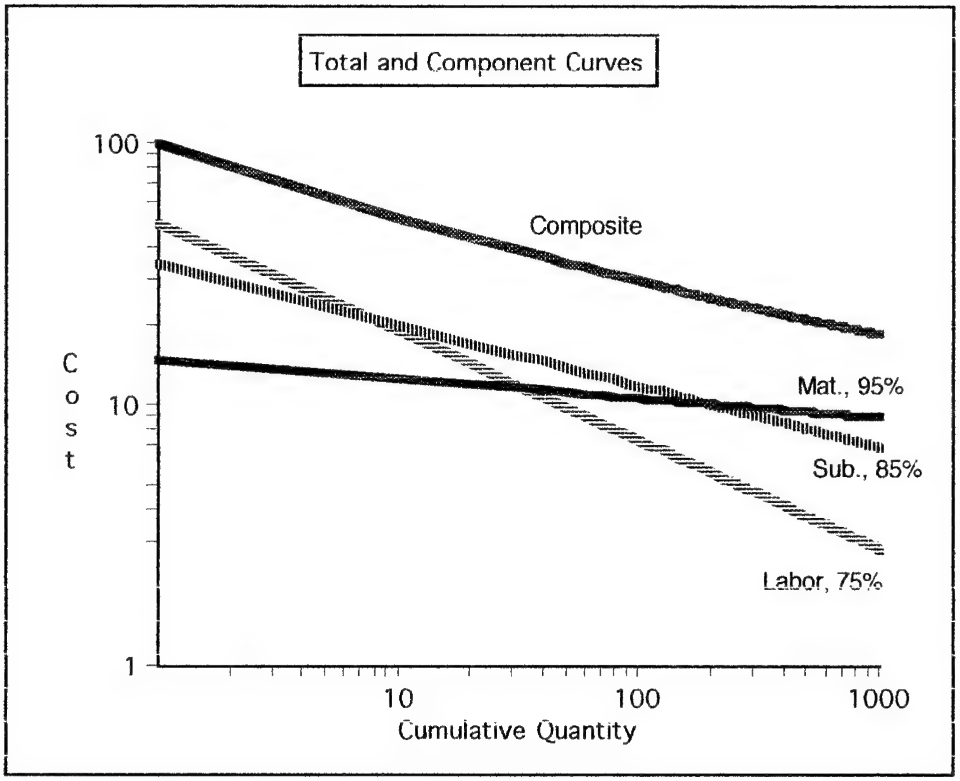

Figure 12. Total and Component Curves

Although labor initially has a higher cost than subcontract items or raw material, it ultimately has the lowest cost because of its steep learning curve. The opposite holds for raw material. Because the costs that are on a flatter learning curve become a larger proportion of total cost, one might expect the composite learning curve to become flatter and bowed.

The composite curve is only slightly bowed in Figure 12. It may seem surprising that the variation in component curves does not produce a greater curvature of the composite curve. The explanation is found in the compression of the Y axis due to the logarithmic scale.

Regression analysis was used to fit a learning curve to the total cost curve. The composite curve has an 85.7% learning slope. The analysis of variance table for the regression is shown in Table 3.

Table 3. ANOVA Table, Composite Curve

Dependent variable is: LCost
No Selector
R squared = 99.2% R squared (adjusted) = 99.2%
s = 0.0213 with 101 - 2 = 99 degrees of freedom

Source	Sum of Squares	df	Mean Square	F-ratio
Regression	5.95596	1	5.95596	13094
Residual	0.045031	99	0.000455	

Variable	Coefficient	s.e. of Coeff	t-ratio	prob
Constant	4.44904	0.0116	382	≤ 0.0001
LQty	-0.222598	0.0019	-114	≤ 0.0001

The fitted relationship is very strong. The cumulative quantity variable is statistically significant at the 99.99% level. The coefficient of determination, R^2, indicates that the regression equation explains more than 99% of the variation of ln(cost). The fitted regression line is very close to the composite line.

Large residuals, differences between actual and predicted cost, could indicate that it is inappropriate to fit a linear composite curve. Residual values were computed in dollars, rather than the logarithm of dollars. These were divided by actual cost and multiplied by 100 to obtain the error expressed as a percentage. Using the absolute value of all errors, the mean absolute error is 1.38%. There was considerable error in predicting the theoretical first unit

cost: 14%. However T1 values tend to be volatile in learning curve analysis due to the high initial steepness of the learning curve. All other estimates were much closer to actual values. Excluding unit 1, the maximum error is 2.47%.

In reality, the analyst does not start with equations and smooth curves. Rather, data exists for each production lot. There would be some random variation for each element of cost: labor, subcontract items, raw materials, and overhead. Due to these variations, neither the component curves nor the composite curve will fit the actual data exactly. Statistical measures such as R^2, the F-statistic, and the b coefficient P-value, are normally far less than shown in Figure 12.

The predicted bowing of the composite curve does not appear to be a sufficient reason for rejecting the common practice of developing composite curves. The answer to research question 1 appears to be that, it is valid to fit composite learning curves.

Analysis of Weapon System Cost History

The cost history of three weapon systems will be analyzed to address research questions two, three and four, reproduced below:

Question 2: Can the addition of a production rate variable improve the learning curve equation?

Question 3: Can the addition of a categorical variable improve learning or, learning and rate, equations?

Question 4: Can a categorical variable provide useful diagnostic information?

Efforts to obtain data on additional weapon systems were not fruitful. Consequently the author was required to use recurring production cost data previously obtained through work.

Multiple Launch Rocket System (MLRS). The MLRS Project Management Office provided cost and schedule data on eleven contracts. The data included monthly deliveries, useful in determining a rate variable. The time-span of deliveries ranged from 5 to 14 months. The rate variable was determined by dividing contract quantity by months of deliveries. There were some months when deliveries were made for more than one contract. In some months no deliveries were made. One large contract completely overlapped a very small contract. These were combined for purposes of rate and learning. The value of the categorical variable was determined based upon the characteristics of the larger contract. This reduced the number of data points to 10.

Fitting a learning curve, $Y = A X^b$, to the data provides good results. The F-ratio for the estimated equation is 92.4 which is highly significant. The coefficient of determination, R^2, adjusted for degrees of freedom is 91.0%. This indicates that 91% of the variation about the mean of ln(Y) is explained by the regression equation. The standard error of the estimate, SEE, is .1338.

Coefficients are of the expected sign; A is positive and b is negative. The value of b implies a learning slope with a reasonable magnitude. The P-value for b is less than .0001, which indicates that b is highly significant.

Adding a rate term results in the equation $Y= A X^b R^f$ and improves the fitted relationship. The F-test increases to 136. R^2 adj. increases to 96.8%. The SEE drops to .0804.

The logarithm of quantity and the logarithm of rate are highly correlated. The correlation coefficient, r, for these two variables is .946. This suggests that there may be problems due to multicollinearity. The regression algorithm may have difficulty ascertaining the separate effect of rate and learning. Also, p-values for coefficients may understate their true statistical significance.

All coefficients are of the expected sign and of a reasonable magnitude. The learning slope becomes less steep with the addition of rate. Some of the reduction of cost is now ascribed to rate. The coefficient f is highly significant, P-value = .0062. However the P-value for b falls to .1704. The rate augmented learning curve appears highly suitable for estimating.

Table 4. MLRS: Comparing Models

Equation	Goodness of Fit		
	F-Ratio	R^2 adj.	SEE
1. $Y = A X^b$	92.4	91.0%	.1338
2. $Y = A X^b R^f$	136.0	96.8%	.0804

Early contracts were single year contracts, while later contracts were multiyear contracts. Given our prior theoretical discussion, it makes sense to consider a downward shift of the cost, quantity, rate surface due to expected savings from multiyear procurement. The estimating equation is specified as $Y = A e^{cZ} X^b R^f$. Z is given a value of zero for single year contracts and a value of 1 for multiyear contracts.

When we fit this equation, the F-ratio falls to 88.5, although it is still highly significant. R^2 adj. is reduced marginally to 96.7%. The SEE is slightly better with a value of .0814. All coefficients are of the expected sign. However the learning slope becomes very flat. Savings attributable to the use of multiyear procurement, adjusted for learning and rate, are estimated at 10.3% which is very plausible. The coefficients b and c are not statistically significant. Although the apparent savings on multiyear procurement is encouraging, this specification may be no better that the one previously discussed.

As a result of further inspection of the data, it was discovered that all single year contracts were also low rate initial production (LRIP) contracts. LRIP is designed to further resolve design and producibility issues. As

discussed previously, greater preparation for production should reduce the theoretical first unit cost and flatten the learning curve slope. One might expect regular production contracts to have a lower estimated first unit cost and a flatter learning slope compared to LRIP contracts, a shift and rotation of the learning curve. The magnitude of the shift should increase due to the simultaneous change to multiyear production. This suggests use of the equation $Y = A\, e^{cZ} X^{(b+dZ)} R^f$.

When this is estimated the F-ratio increases to 143. R^2 adj. increases to 98.4% and SEE drops noticeably to .0558. All coefficients are significant. All coefficients have the expected sign. However "d" is large. As a result, (b+d) is just slightly greater than one. This implies a learning slope that is a little over 100% in the regime characterized by multiyear contracts and regular production. This slope is probably mistaken and is caused by the high correlation between rate and quantity. Except for this fly in the ointment, this model appears to be a very successful explanation of the factors present in the cost history of this system. It is considered superior to the equation previously discussed. Results to this point are summarized in Table 5.

Table 5. MLRS: Comparing Models - 2

Equation	Goodness of Fit		
	F-Ratio	R^2 adj.	SEE
1. $Y = A\, X^b$	92.4	91.0%	.1338
2. $Y = A\, X^b R^f$	136.0	96.8%	.0804
3. $Y = A\, e^{cZ} X^b R^f$	88.5	96.7%	.0814
4. $Y = A\, e^{cZ} X^{(b+dZ)} R^f$	143.0	98.4%	.0558

The most successful specifications found in Table 5 are equations 2 and 4. For the most part, equation 4, which allows for a shift and rotation of the learning curve based upon regime, is superior. However it results in a learning slope for $regime_1$ that is slightly over 100%, which seems unlikely, and is probably due to the high correlation between rate and quantity. Consequently it may be a matter of judgment whether one prefers equation 2 or 4.

It is possible that the change to regular production entailed more automation. Given prior discussion in Chapter III, this would imply a change in the rate slope. This can be captured by the equation $Y = A\, e^{cZ} X^{(b+dZ)} R^{(f+gZ)}$. The use of this equation produces estimated coefficients identical to those obtained by dividing the data set by regime and fitting the rate augmented learning curve equation, $Y = A\, X^b R^f$, to each regime.

A benefit of the proposed equation, $Y = A\, e^{cZ} X^{(b+dZ)} R^{(f+gZ)}$, is that statistical tests can be obtained for changes in the intercept, learning slope and rate slope. When the equation is fitted, the F test, R^2 adj., and SEE were comparable to that of the model previously discussed. However the learning slope for $regime_1$ was of the wrong sign and of an unacceptable magnitude. The change in intercept and change in rate slope were not statistically significant. This specification did not prove successful.

Because the shift was not statistically significant in the preceding model, one could consider dropping e^{cZ} from the model. However, it is generally recommended that analysts retain intercept terms (Neter, 1990:163) even when they are not statistically significant. This prescription probably holds for a change in intercept term as well. As an experiment, the intercept term was dropped and the equation $Y = A\,X^{(b+dZ)}\,R^{(f+gZ)}$ was estimated. Many results appear stellar: F-ratio = 223, R^2 adj. = 99.0%, SEE = .0448. Moreover all coefficients were statistically significant. However, as was the case in the previous model, (b+d) has the wrong sign and is of a high magnitude. This specification is not recommended.

Patriot Missile. The Patriot Project management office provided cost and quantity data. In the absence of schedule information, annual quantity is used as a proxy for rate. There are six data points.

The standard learning curve equation fit the data well. The F-ratio is 127 which is highly significant. R^2 adj. is 96.2%. The SEE equals .0404. All coefficients are of the proper sign, significant, and of reasonable magnitude.

The first three data points are the analog model. The last three are the digital model. Army management asked if there was a change in learning curve slopes between models. The data could be broken in two and each data set estimated separately. Chance variation in the data would suggest the likelihood that a different first unit cost and learning slope would be estimated. The same coefficients are estimated using a categorical variable and

the equation $Y = A\ e^{cZ} X^{(b+dZ)}$, which enables a shift and rotation of the learning curve. The added benefit of this method is that the statistical significance of any change in intercept and slope can be evaluated.

Estimating $Y = A\ e^{cZ} X^{(b+dZ)}$ provides the following result. The F ratio drops greatly to 51.1. There is a very small increase in R^2 adj. to 96.8%. The SEE decreases slightly to .0370. The c and d coefficients are not statistically significant. Their p-values are .7700 and .8875 respectively. The change in slope and intercept between the analog and digital models is not statistically significant. The shift and rotation appear to be due to chance.

The rate augmented learning curve, $Y= A\ X^b\ R^f$, results in an improved fit of the data compared to the standard learning curve. However the high correlation between ln(quantity) and ln(rate), .829, indicates that there may be problems due to multicollinearity. The F-ratio increases to 141.0. R^2 adj. increases to 98.2%, and the SEE falls to .0273. All coefficients are of the expected sign and of reasonable magnitude. All coefficients have acceptable levels of statistical significance. The p-values for b and f are .0052 and .1016 respectively. Results are summarized in Table 6.

Table 6. Patriot: Comparing Models

Equation	Goodness of Fit		
	F-Ratio	R^2 adj.	SEE
1. $Y = A\ X^b$	127.0	96.2%	.0404
2. $Y = A\ e^{cZ} X^{(b+dZ)}$	51.1	96.8%	.0370
3. $Y= A\ X^b\ R^f$	141.0	98.2%	.0273

Advanced Medium Range Air-to-Air Missile (AMRAAM). This data set was previously obtained using the InfoArch data base. Detailed schedule information was not available. Contract quantity was used as a proxy for rate. The author was unaware of any changes in regime, therefore no categorical variable was used. The cost history consisted of seven data points.

The standard learning curve equation fit the data extremely well. The F-ratio is 309. R^2 adj. is 98.1%. The SEE is .0897. All estimated coefficients are of the proper sign, of reasonable magnitude, and highly significant.

There is a very high degree of correlation between ln(quantity) and ln(rate), .976, which suggests that there may be problems due to multicollinearity. When the rate variable is added, the F-ratio drops to 178 and R^2 adj. and SEE show very little improvement. The coefficient for rate has the wrong sign and is of a high magnitude. The learning slope becomes very steep. It appears that the regression algorithm cannot sort out the separate contributions of rate and quantity. In this case, multicollinearity has led to a significant problem, the misestimation of coefficients b and f.

In this instance the addition of a rate variable did not improve results. Adding another regressor does not guarantee an improved relationship.

Table 7. AMRAAM: Comparing Models

Equation	Goodness of Fit			
	F-Ratio	R^2 adj.	SEE	
1. $Y = A X^b$	309.0	98.1%	.0897	
2. $Y = A X^b R^f$	178.0	98.3%	.0839	Wrong sign for rate

An Alternative Approach to Rate and Quantity

The rate augmented learning curve equation used up to this point is $Y = A X^b R^c$. A possible alternative, broached in Chapter III, is to start with a standard U-shaped cost curve based upon rate, and add a learning variable. This can be described as a learning augmented rate curve, or as a learning augmented U-shaped cost curve. One can use equation 27, $Y = A + bR + cR^2 + dLnX$. These are two different ways of specifying equations that conform to the general equation presented in Chapter I: (3) Y = f(cumulative quantity, production rate). Other specific equations are possible. Research question number 5 is reproduced below:

> Question 5: Should new equations of learning and rate be explored?

Fitting the equation $Y = A + bR + cR^2 + dLnX$ to the MLRS system provides an F test that is very significant at 160. R^2 adj. is 98.2%. All coefficients are of the expected sign and are highly significant. The predicted minimum cost based upon rate is within the range of the data set for rate. The rate associated with minimum cost is somewhat higher than the midpoint of the range of rate.

Fitting the learning augmented rate curve, $Y = A + bR + cR^2 + dLnX$, to Patriot results in a very significant F ratio, 157. R^2 adj. is 98.9%. All estimated coefficients are of the expected signs. P-values for the b, c, and d coefficients are .0990, .1037, and .0090, respectively. The predicted minimum cost rate is

somewhat higher than the midpoint of the range of rate.

In the case of AMRAAM, the F ratio is highly significant with a value of 327. R^2 adj. is 99.4%. All coefficients are of the expected sign and are highly significant. Once again the predicted minimum is somewhat higher than the midpoint of the range.

The standard error of the estimate, SEE, was used to compare the learning augmented rate curve, $Y = A + bR + cR^2 + d\text{Ln}X$, with the standard rate augmented learning curve, $Y = A X^b R^f$. In the former case, the SEE is based on Y. In the latter case, the SEE is based upon ln(Y). In order to obtain comparable values of SEE, SEE was computed in terms of Y for both cases. SEE is equal to the root mean squared error. Values of Y were estimated based upon the fitted equation $Y = A X^b R^f$. These estimates were compared to actual values of Y to obtain error terms which were then squared and summed. The resulting value was divided by the degrees of freedom associated with the regression equation $Y = A X^b R^f$. The square root was then taken. Comparisons of standard error of the estimate, in Y, are shown in Table 8 for both models.

Table 8. Standard and Alternative Cost, Quantity, Rate Relationships

Equation	Standard Error of the Estimate (SEE)		
	MLRS	Patriot	AMRAAM
1. $Y = A X^b R^f$	463.0	18.45	.110
2. $Y = A + bR + cR^2 + d\text{Ln}X$	410.6	7.93	.039

The standard error of the estimate, SEE, is lower using the learning augmented rate curve, equation 2 in Table 8, for these three systems. Although one would expect some reduction in SEE due to the presence of an additional coefficient, the comparison appears favorable to equation 2.

Based upon the F-test, R^2 adj. and SEE, the learning augmented rate curve provides a very good fit of the data for the three systems examined. All estimated coefficients have the expected sign and have an acceptable level of significance. For each weapon system, the predicted economic production rate is within the range of the data set. The statistical results are encouraging. However these findings remain tentative due to the small number of systems, and the limited number of data points for the systems examined.

Combining the discussion on the impact of learning and rate contained in Chapter II, with the discussion of equation 27, $Y = A + bR + cR^2 + dLnX$, found in Chapter III indicates that the learning augmented rate curve has an adequate theoretical basis.

Combining theoretical discussion with the statistical analysis suggests that an affirmative answer to research question 5 is warranted; new equations of learning and rate should be explored.

V. Findings and Conclusions

Chapter I introduced the following general research question: What is the proper identification of variables and specification of equations to estimate unit production cost? This thesis has attempted to address this question thoroughly. The three major variables identified are cumulative quantity, production rate, and any major change in the production regime of a weapon system.

A standard method of incorporating regime based upon the use of a categorical variable was developed in Chapter III. In a learning curve model, a change in regime can lead to either a shift, or a shift and rotation, of the learning curve. There are more options in a rate augmented learning model. A change in regime can lead to a shift, a shift and a learning rotation, a shift and a rate rotation, or a shift with both learning and rate rotations, of the cost-quantity-rate surface. Specific research questions are addressed below:

> Question 1: Is it valid to fit a linear composite learning curve given that component learning curves are linear and of varying slopes?

A hypothetical, representative case was developed to answer this question. Although there was a bowing of the composite learning curve as predicted by several authors, the bowing was minor and well within the estimating error of learning curve estimates. Hence question 1 can be answered affirmatively.

Question 2: Can the addition of a production rate variable improve the learning curve equation?

Improvement was evaluated in terms of individual coefficients and the model as whole. Individual coefficients were evaluated based upon their sign, magnitude, and statistical significance. Models were evaluated in terms of their F test, coefficient of determination adjusted for degrees of freedom, and the standard error of the estimate. The addition of a production rate variable led to significant improvement for the Multiple Launch Rocket System (MLRS) and the Patriot missile system. Adding a production rate variable did not produce desirable results in the case of the Advanced Medium Range Air-to-Air Missile (AMRAAM) system. Production rate appears to be an important variable. However due to the correlation between cumulative quantity and production rate, which commonly occurs, the regression algorithm may have difficulty separating the effects of quantity and rate. Hence judgment is required in evaluating a rate augmented learning curve model. The answer to question two is affirmative, but qualified. The addition of a production rate variable can, but need not, improve the learning equation.

Question 3: Can the addition of a categorical variable improve learning or, learning and rate, equations?

This question could only be examined for two systems MLRS and Patriot, since no reason for using a categorical variable was apparent for AMRAAM. In the case of MLRS, the addition of a categorical variable improved the fit of the data compared to either the learning curve equation, or the rate augmented learning curve equation. The resulting equation was generally logical and good. However it contained one flaw which is probably due to the correlation of quantity and rate. Hence it is a matter of judgment whether the equation with the categorical variable is superior to the rate augmented learning curve. In the case of Patriot, the addition of a categorical variable did not lead to an improved equation. However it did provide valuable diagnostic information. The author is confident that the correct answer to question 3 is "yes". This study provided some support for an affirmative answer but the support is weak.

Question 4: Can a categorical variable provide useful diagnostic information?

The answer to this question is a definite "yes." In the case of Patriot, the use of a categorical variable made it possible to determine that a shift and rotation of the learning curve was not statistically significant and was probably due to chance. In the case of MLRS, examination of diagnostic information associated with the categorical variable provided numerous insights.

Question 5: Should new equations of learning and rate be explored?

There may be many plausible equations involving learning and rate that could be explored. A possible deficiency of the standard rate augmented learning curve is that it does not generate increasing costs at high production rates. This study introduced a new equation that does generate higher costs at higher production rates, consistent with standard economic theory. The equation is logical and fits the data very well for all three weapon systems examined. This further suggests an affirmative answer to question five.

This study is based upon three weapon systems. MLRS had ten data points. Patriot had six and AMRAAM had seven. More systems and production lots should be examined to further answer questions two through five. In particular questions two, three, and the new proposed model of rate and learning contained in the discussion of question five, could benefit from the study of additional weapon systems and systems with more production lots. None of the systems examined experienced a reduction in production rate. It is expected that such a reduction would reduce the multicollinearity between cumulative quantity and rate, and might improve analysis.

The research questions and answers in this thesis apply at the level of recurring production cost. This often includes labor, subcontract items, raw material, and several overhead costs. The findings contained in this study are most applicable at that level and may not apply when estimating at a lower level.

Appendix 1, Army Cost Element Structure

Cost Element Structure

1.0 RESEARCH, DEVELOPMENT, TEST, AND EVALUATION (RDT&E)-FUNDED ELEMENTS

1.01 DEVELOPMENT ENGINEERING*
1.02 PRODUCIBILITY ENGINEERING AND PLANNING (PEP)*
1.03 DEVELOPMENT TOOLING*
1.04 PROTOTYPE MANUFACTURING*
1.05 SYSTEM ENGINEERING/PROGRAM MANAGEMENT
1.051 PROJECT MANAGEMENT ADMINISTRATION (PM CIV/MIL)
1.052 OTHER
1.06 SYSTEM TEST AND EVALUATION
1.07 TRAINING
1.08 DATA
1.09 SUPPORT EQUIPMENT
1.091 PECULIAR
1.092 COMMON
1.10 DEVELOPMENT FACILITIES
1.11 OTHER RDT&E

2.0 PROCUREMENT-FUNDED ELEMENTS

2.01 NONRECURRING PRODUCTION
2.011 INITIAL PRODUCTION FACILITIES (IPFs)*
2.012 PRODUCTION BASE SUPPORT (PBS)*
2.013 OTHER NONRECURRING PRODUCTION*
2.02 RECURRING PRODUCTION
2.021 MANUFACTURING*
2.022 RECURRING ENGINEERING*
2.023 SUSTAINING TOOLING*
2.024 QUALITY CONTROL*
2.025 OTHER RECURRING PRODUCTION*
2.03 ENGINEERING CHANGES*
2.04 SYSTEM ENGINEERING/PROGRAM MANAGEMENT
2.041 PROJECT MANAGEMENT ADMINISTRATION (PM CIV/MIL)
2.042 OTHER
2.05 SYSTEM TEST AND EVALUATION, PRODUCTION
2.06 TRAINING
2.07 DATA
2.08 SUPPORT EQUIPMENT
2.081 PECULIAR
2.082 COMMON

* These elements should be further subdivided to reflect the MIL-STD-881B Level 3 prime mission equipment WBS elements. Greater level of detail is permissible.

2.09 OPERATIONAL/SITE ACTIVATION
2.10 FIELDING
2.101 INITIAL DEPOT-LEVEL REPARABLES (SPARES)
2.102 INITIAL CONSUMABLES (REPAIR PARTS)
2.103 INITIAL SUPPORT EQUIPMENT
2.104 TRANSPORTATION (EQUIPMENT TO UNIT)

2.105 NEW EQUIPMENT TRAINING (NET)
2.106 CONTRACTOR LOGISTICS SUPPORT
2.11 TRAINING AMMUNITION/MISSILES
2.12 WAR RESERVE AMMUNITION/MISSILES
2.13 MODIFICATIONS
2.14 OTHER PROCUREMENT

3.0 MILITARY CONSTRUCTION (MC)-FUNDED ELEMENTS
3.01 DEVELOPMENT CONSTRUCTION
3.02 PRODUCTION CONSTRUCTION
3.03 OPERATIONAL/SITE ACTIVATION CONSTRUCTION
3.04 OTHER MC

4.0 MILITARY PERSONNEL (MP) DIRECT-FUNDED ELEMENTS (not reimbursed by any other appropriation)
4.01 CREW
4.02 MAINTENANCE (MTOE)
4.03 SYSTEM-SPECIFIC SUPPORT
4.04 SYSTEM ENGINEERING/PROGRAM MANAGEMENT
4.041 PROJECT MANAGEMENT ADMINISTRATION (PM MIL)
4.042 OTHER
4.05 REPLACEMENT PERSONNEL
4.051 TRAINING
4.052 PERMANENT CHANGE OF STATION (PCS)
4.06 OTHER MP

5.0 OPERATIONS AND MAINTENANCE (O&M)-FUNDED ELEMENTS
5.01 FIELD MAINTENANCE CIVILIAN LABOR**
5.02 SYSTEM-SPECIFIC BASE OPERATIONS
5.03 REPLENISHMENT DEPOT-LEVEL REPARABLES (SPARES)**
5.04 REPLENISHMENT CONSUMABLES (REPAIR PARTS)**
5.05 PETROLEUM, OIL, AND LUBRICANTS (POL)**
5.06 END-ITEM SUPPLY AND MAINTENANCE
5.061 OVERHAUL (P7M)
5.062 INTEGRATED MATERIEL MANAGEMENT
5.063 SUPPLY DEPOT SUPPORT
5.064 INDUSTRIAL READINESS
5.065 DEMILITARIZATION
5.07 TRANSPORTATION
5.08 SOFTWARE

** These elements should be further subdivided to reflect the MIL-STD-881B Level 2 prime mission equipment WBS elements and the support equipment element. Greater level of detail is permissible.

5.09 SYSTEM TEST AND EVALUATION, OPERATIONAL
5.10 SYSTEM ENGINEERING/PROGRAM MANAGEMENT
5.101 PROJECT MANAGEMENT ADMINISTRATION (PM CIV)
5.102 OTHER
5.11 TRAINING
5.12 OTHER O&M

6.0 DEFENSE BUSINESS OPERATIONS FUND (DBOF) ELEMENT
6.01 DBOF CLASS IX WAR RESERVES

Appendix 2, Cost Definitions

2.01 NONRECURRING PRODUCTION

2.011 INITIAL PRODUCTION FACILITIES (IPFs)
This element includes the cost of the initial hard tooling and production line set up to support low-rate and full-scale production of the system; and the cost of fabrication, assembly, and installation of tools (including modification and rework of development tools for production purposes), dies, templates, patterns, form block manufacture, jigs, fixtures, master forms, inspection equipment, handling equipment, load bars, work platforms (including installation of utilities thereon), and test equipment (such as checkers and analyzers) to support the manufacture of the specified system. It includes initial and duplicate sets of tools necessary to reach full-rate production plus modification of LRIP tool records, establishment of make-or-buy and manufacturing plans on nonrecurring tools and equipment, scheduling and control of tool orders, and programming and preparation of software for numerically controlled machine equipment. Included in this element are any provision of industrial facilities (PIF), depot maintenance plant equipment (DMPE), and layaway of industrial facilities that are system specific.

2.012 PRODUCTION BASE SUPPORT (PBS)
This element includes the procurement-funded costs of construction, conversion, or expansion of facilities for production, inventory, or maintenance required to accomplish the program. These costs may be identified with either or both the contractor and in-house efforts. They may be identified with the total system or with specific components of the total system, such as the engine. This element excludes any PIF costs included in IPFs.

2.013 OTHER NONRECURRING PRODUCTION
This element includes any procurement-funded, nonrecurring production costs not included in the above subelements. Costs must be system specific and clearly identified. For example, disposal, demilitarization, or layaway costs of Government-owned production equipment should be included here as a cost to the system.

2.02 RECURRING PRODUCTION

2.021 MANUFACTURING
This element includes the costs of material, labor, and other expenses incurred in the fabrication, checkout, and processing of parts, subassemblies, and major assemblies/subsystems needed for the final system. This element also includes Government-furnished equipment and material, as well as costs of subcontractors and purchased parts/equipment. The element further includes costs of the efforts to integrate and assemble the various subassemblies into a working system, costs to install special and general equipment, costs to paint and package the system for shipment to its acceptance destination, and costs associated with preplanned product improvements. It also includes moves in order to assemble into a final system.

2.022 RECURRING ENGINEERING
This element includes the costs of all engineering efforts performed in support of production, including maintainability/reliability engineering, maintenance engineering, value engineering, and production engineering costs associated with the system. It also includes redesign, evaluation, and other support engineering efforts (either in-house, contract, or separate contractor) directly involved with production of the components/end item, e.g., maintenance of the TDP, preparation of engineering change proposals (ECPs), engineering change orders (ECOs), and analysis of test results.

2.023 SUSTAINING TOOLING

This element includes the costs of maintenance replacement or modification of tools and test equipment after the start of production. It includes the replacement of initial tools that break down, and modification, maintenance, and rework of initial and duplicate sets of tools occurring after production begins.

2.024 QUALITY CONTROL

This element includes the costs of implementing controls necessary to ensure that a manufacturing process produces a system that meets the prescribed standards. Included are costs of receiving, in-process, and final inspections of tools, parts, subassemblies, and complete assemblies. It also includes such tasks as reliability testing, establishment of acceptable quality levels (AQLs), statistical methods for determining performance of manufacturing processes, preparation and review of reports relating to these tasks, stockpile reliability testing, and the performance of production acceptance tests (PATs).

2.025 OTHER RECURRING PRODUCTION

This element includes any procurement-funded, recurring production costs not included in the above subelements. Costs must be system specific and clearly identified, e.g., warranty cost for a specific item.

Appendix 3, Derivation of Midpoint Equations

Exact derivation of lot midpoint:

$Y=AX^b$

Total cost = $\sum Y$ = the sum from X=F to X=L of AX^b

Average cost = the sum from X=F to X=L of AX^b / (L-F+1)

Average cost = AM^b

Therefore: AM^b = the sum from X=F to X=L of AX^b / (L-F+1)

Dividing both sides by A results in

M^b = the sum from X=F to X=L of X^b / (L-F+1)

Taking the bth root of both sides results in

M = (the sum from X=F to X=L of X^b / (L-F+1)) ^(1/b)

Which is equation 8:

(8) M = (the sum from X=F to X=L of X^b/(L-F-1))^(1/b)

Derivation of approximation 1, equation (9):

$Y = AX^b$

Total cost is the area underneath the learning curve. This is equal to the definite integral from F to L of AX^b

Total cost = (A/(b+1) $L^{(b+1)}$) - (A/(b+1) $F^{(b+1)}$) = (A/(b+1)) ($L^{(b+1)} - F^{(b+1)}$)

To obtain average cost, divide by the number of units in the lot, (L-F+1)

Average cost = (A/ ((b+1)(L-F+1))) ($L^{(b+1)} - F^{(b+1)}$)

Average cost = AM^b

Therefore

AM^b = (A/ ((b+1)(L-F+1))) ($L^{(b+1)} - F^{(b+1)}$)

Dividing both sides by A results in

M^b = ($L^{(b+1)} - F^{(b+1)}$) / ((b+1) (L-F+1))

Taking the bth root of both sides

M = (($L^{(b+1)} - F^{(b+1)}$) / ((b+1) (L-F+1))) ^ (1/b)

However the learning curve is a not a continuous function. Therefore the endpoints must be adjusted by .5 units as follows:

M = ((($(L+.5)^{(b+1)} - (F-.5)^{(b+1)}$) / ((b+1) (L-F+1))) ^ (1/b)

Which is equation 9:

(9) M = (((L+.5)^(b+1)-(F-.5)^(b+1))/((b+1)*LotSize)) ^ (1/b)

Equation 9 is equivalent to the equation found on page 40 of Alpha & Omega and the Experience Curve, United States Army Missile Command, Huntsville, Alabama (1965)

Appendix 4, Exact Midpoint Calculation

This Excel Macro function provides an exact calculation of the lot midpoint for a production lot. It has three arguments: first unit in lot, last unit in lot, and learning curve slope. A 90% learning curve slope is entered as .9.

```
ALM
=ARGUMENT("First")
=ARGUMENT("Last")
=ARGUMENT("Slope")
=SET.NAME("K",First)
=SET.NAME("sum",0)
=SET.NAME("LotSize",Last-First+1)
=SET.NAME("b",LN(Slope)/LN(2))
=IF(K=Last+1,GOTO(A13))
=SET.NAME("sum",sum+K^b)
=SET.NAME("K",K+1)
=GOTO(A9)
=(sum/LotSize)^(1/b)
=RETURN(A13)
```

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Vita

Mark W. Glenn was born in Washington, D.C. in 1953. He graduated from the University of Delaware in Newark, Delaware in 1975 having earned a Bachelor of Science degree in Economics. He entered government service in 1976 and worked in the logistics career field at the U.S. Army Aviation Systems Command in St. Louis, Missouri. He completed his Master of Arts degree in Economics from the University of Delaware in 1979 having achieved the highest grade in his class on the Master's comprehensive exams. He became an operations research analyst at the Aviation Systems Command in 1980. He supported source selections, contract negotiations, and developed cost estimates for major aviation systems. He moved to Washington, D.C. in 1985 working first for the U.S. Army Cost and Economic Analysis Center, and then for Headquarters U.S. Army Materiel Command. In 1989, he moved to Huntsville, Alabama where his primary duty was to develop cost estimating relationships for Army Missile systems. He married the former Jean Louise Mulcahy in 1974. Their daughter, Melody, was born in 1985. Upon completion of his Master of Science degree in Cost Analysis from the Air Force Institute of Technology in 1997, he will return to Alabama and work for the U.S. Army Space and Strategic Defense Command.

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13. ABSTRACT (*Maximum 200 Words*)

This research identifies variables and specifies equations that can be used to estimate the unit production cost of a weapon system. It is concerned with both explanation and prediction. Three major variables identified are cumulative quantity, production rate, and change in regime. Cumulative quantity is used in learning curve theory. Production rate is found in the U-shaped short and long run cost curves of economic theory. This study uses the term regime to refer to any major change in the production environment of a weapon system. This research attempts to integrate the use of these three variables.

A change in regime may be due to a change in acquisition strategy, configuration, or manufacturing method. It is recommended that a categorical variable be used to capture the effect of a change in regime. Several specific equations are proposed and discussed. In general, they entail a shift, shift and rotation, or shift and two rotations of the cost-quantity-rate surface due to a change in regime. Many accepted methods of integrating learning and rate do not produce U-shaped rate curves; this study suggests one that does. Principles and equations discussed are applied in modeling the cost history of three missile systems.

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